

Forecast performance of robust multivariate range-based GARCH models: Evidence from oil, gold, equity, and Bitcoin markets

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ABSTRACT

We propose robust multivariate range-based volatility models designed to improve the forecasting of risk and interdependencies across financial markets. Methodologically, the paper combines two complementary features within a single multivariate framework: robustness to outliers and the use of range-based volatility measures constructed from daily open, high, low, and close prices. As a result, the proposed framework operates entirely in a daily-data setting and does not require high-frequency intraday observations. Two new model specifications are introduced, both extending the standard dynamic conditional correlation framework to enhance model stability and predictive accuracy. Empirical results indicate that the proposed models provide more reliable forecasts of covariances and portfolio risk than conventional multivariate volatility models. They also deliver more accurate Value-at-Risk forecasts, underscoring the practical relevance of robust range-based volatility models for financial forecasting and risk management. The empirical analysis covers crude oil, gold, equities, and Bitcoin, reflecting the growing integration of traditional and alternative asset classes.

1. Introduction

Modeling relationships among financial time series is of considerable practical importance. Understanding and forecasting volatility and cross-market dependencies are central to modern financial analysis. Accurate covariance and correlation forecasts are essential for portfolio allocation, hedging strategies, and systemic risk monitoring. These issues have become particularly important in recent years due to the increasing integration of traditional assets, such as equities, commodities, and gold, with alternative assets, including cryptocurrencies. Consequently, financial institutions, portfolio managers, and regulators increasingly rely on multivariate volatility models to assess portfolio risk and tail exposures across heterogeneous asset classes.

Multivariate GARCH models are particularly well suited to modeling financial time series because they allow for the joint modeling of time-varying volatilities and correlations across assets, which is crucial for portfolio allocation, hedging strategies, and risk management. Unlike many alternative approaches, these models provide a coherent framework for generating dynamic covariance matrices that can be directly employed in portfolio optimization and Value-at-Risk (VaR) calculations. As a result, multivariate GARCH specifications have become the

dominant econometric framework for modeling volatility interactions across financial and commodity markets (see, e.g., Ali & Khurram, 2025; Archakov et al., 2026; Lai et al., 2025; Yin et al., 2025; Campos-Martins & Amado, 2026; Kim, 2026).

A substantial strand of the volatility forecasting literature relies on high-frequency data and realized volatility measures, often modeled within frameworks such as the HAR model (Corsi, 2009). Although these approaches can provide statistically more efficient volatility estimators, their practical implementation requires access to high-frequency intraday data and involves several methodological choices, including the sampling frequency, the selection of realized volatility estimators, and the treatment of market microstructure noise. In multivariate settings, the construction of realized covariance measures introduces additional challenges related to nonsynchronous trading and computational complexity.

In contrast, the approach adopted in this paper focuses on volatility models based on daily data. Range-based estimators constructed from daily open, high, low, and close (OHLC) prices provide substantially more information than close-to-close returns while avoiding the data requirements and implementation challenges associated with high-frequency approaches. The empirical illustration presented in this

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study is based on four assets representing major and economically distinct segments of global financial markets: crude oil, gold, equities, and Bitcoin. These assets are among the most widely traded instruments within their respective classes and are frequently used as benchmarks in empirical financial research. At the same time, they exhibit markedly different volatility dynamics, market structures, and responses to macroeconomic shocks. Oil reflects commodity and energy markets, gold is traditionally viewed as a safe-haven asset, equities represent the core of financial markets and global economic activity, while Bitcoin captures the rapidly evolving cryptocurrency market characterized by extreme volatility and structural breaks. Such a heterogeneous set of assets provides an appropriate environment for evaluating multivariate volatility models, as it facilitates the analysis of cross-market volatility interactions and risk transmission across fundamentally different asset classes.

The rapid proliferation of volatility models has created a nontrivial challenge for empirical research, making it increasingly difficult to determine which modeling mechanisms are most effective in improving volatility and covariance forecasts in practice. Recent studies have emphasized the importance of two aspects of financial volatility modeling: (1) robustness to extreme observations and (2) the use of statistically more efficient volatility estimators than close-to-close returns, particularly during periods of heightened market uncertainty. Nevertheless, these two strands of the literature have largely evolved independently, and their joint empirical relevance in multivariate settings has remained unexplored.

Financial and commodity prices are well known to exhibit sudden jumps and extreme observations, especially during periods of market stress. A prominent example is the oil market, which experienced severe disruptions following the outbreak of the COVID-19 pandemic in 2020. It is well established that outliers can severely distort tests of conditional homoscedasticity, parameter estimates, volatility forecasts, and risk measures in univariate frameworks (see [Hotta & Trucíos, 2018](#), for a survey). However, the literature addressing the impact of outliers in multivariate volatility models remains relatively limited. [Grane et al. \(2019\)](#), for instance, developed a wavelet-based outlier detection algorithm applied to residuals from multivariate volatility models and examined its effect on correlations in several popular specifications, including the diagonal BEKK (Baba, Engle, Kraft, and Kroner; [Engle & Kroner, 1995](#)), CCC ([Bollerslev, 1990](#)), and DCC ([Engle, 2002](#)) models. Their Monte Carlo results demonstrate that conditional correlations are highly sensitive to the presence of outliers. Similar conclusions were reached by [LaBarr \(2010\)](#) for the DCC model and by [Boudt et al. \(2013\)](#) and [Trucíos et al. \(2018\)](#) for the corrected DCC (cDCC; [Aielli, 2013](#)) specification.

At the same time, there is strong empirical evidence that large return shocks tend to exert a relatively smaller effect on subsequent volatility than moderate shocks (e.g., [Andersen et al., 2007](#); [Bauwens & Storti, 2009](#); [Boudt et al., 2013](#)). Standard multivariate GARCH models, however, generally fail to capture this feature and often overstate volatility following extreme returns. Consequently, various robust modeling strategies have been proposed to mitigate the excessive influence of extreme observations. [Section 2](#) provides an overview of the relevant literature on robust multivariate volatility models and highlights their advantages during periods of heightened market uncertainty.

In addition to the appropriate treatment of outliers, a practical challenge in modeling financial markets stems from the limited availability of high-quality intraday observations. High-frequency data are often available only for relatively short periods, restricted to the most liquid instruments, or accessible only at considerable cost. As a result, researchers have long relied on alternative volatility measures derived from more readily available data, and interest in these approaches has recently resurged. In this context, range-based models have regained attention because they exploit daily OHLC prices, which are widely available across markets and contain substantially more information about price variability than closing prices alone. A growing body of

empirical evidence demonstrates that range-based volatility measures consistently outperform return-based alternatives in terms of forecasting accuracy (see [Petropoulos et al., 2022](#), for a comprehensive survey).

Taken together, these considerations suggest that both robustness to extreme observations and the use of range-based volatility measures are particularly important in empirical settings characterized by heightened uncertainty and abrupt shifts in volatility. From an applied perspective, a modeling framework that integrates these two features within a unified multivariate structure is therefore highly desirable. Situations involving outliers or unexpected volatility shocks are pervasive in empirical financial research, and the issue becomes even more pronounced when analyzing portfolios composed of heterogeneous assets. In such cases, elevated volatility in a single portfolio component may substantially affect the dynamics of conditional variances and correlations, underscoring the need for models that are both robust and informationally efficient.

This study makes two main contributions to the financial econometrics literature. First, we introduce two robust multivariate range-based volatility models that combine the advantages of range-based specifications with robust estimation techniques. In the first model, we apply the modified bounded innovation propagation (BIP) procedure of [Boudt et al. \(2013\)](#) to the DCC-RGARCH framework of [Fiszeder et al. \(2019\)](#). In the second model, a different approach is adopted in the univariate component, whereby the modified BIP procedure is combined with the robust estimation approach of [Muler and Yohai \(2008\)](#). Accordingly, the second model represents a multivariate extension of the BM-RGARCH univariate volatility model proposed by [Fiszeder et al. \(2024\)](#). Our contribution extends beyond a simple combination of existing modeling elements: we introduce modifications that enable the robust filtering mechanism to be applied to variables inherent in range-based volatility models. Both models employ the DCC parameterization because of its popularity and ease of implementation; however, we adopt the corrected DCC (cDCC) specification proposed by [Aielli \(2013\)](#). To the best of our knowledge, these are the first robust multivariate range-based volatility models proposed in the literature. Unlike existing approaches, the proposed models jointly address two important challenges in financial volatility modeling: sensitivity to extreme observations and the efficient use of intraday price information contained in the high-low range. These models are designed to enhance both the robustness and informational efficiency of multivariate volatility frameworks, which is essential for accurate risk measurement and portfolio management in financial markets.

The second contribution is empirical and directly relevant to financial applications. We demonstrate that the proposed models deliver more accurate forecasts of covariances and Value-at-Risk, two key measures used in portfolio risk management and financial regulation. By applying the models to a diversified portfolio consisting of oil, gold, equities, and Bitcoin, we illustrate how robust range-based volatility models can improve the assessment of cross-asset risk exposures in portfolios that combine traditional and alternative assets. The remainder of the paper is organized as follows. [Section 2](#) reviews the relevant literature. [Section 3](#) introduces the theoretical background and presents the proposed models. [Section 4](#) describes the data and reports summary statistics. [Section 5](#) presents the in-sample estimation results, including parameter estimates and conditional correlations. [Section 6](#) evaluates the accuracy of covariance forecasts for the 30 models under consideration. [Section 7](#) reports the results of the VaR forecasting exercise. [Section 8](#) provides a robustness check based on the Parkinson variance estimator. [Section 9](#) discusses the limitations of the study and directions for future research, and [Section 10](#) concludes.

2. Literature review

Two promising strands of research have recently emerged in the multivariate modeling of financial time series volatility. The first concerns robust modeling and forecasting methods, whereas the second

focuses on the use of range-based volatility estimators. Within the former strand, two main approaches have been proposed to address the problem of outliers in financial data. The first involves iteratively identifying and removing outliers, followed by fitting the model to the cleaned data until no further outliers are detected. Under this approach, extreme observations are identified, corrected, and subsequently analyzed using standard volatility models. The second approach relies on estimators that are inherently robust to outliers. Our focus is on the latter. Accordingly, we review studies that introduce robust estimators for multivariate volatility models and apply them to financial time series. For a broader discussion of related issues, including testing, specification, estimation, and forecasting, see [Hotta and Trucíos \(2018\)](#).

[Boudt and Croux \(2010\)](#) introduced a robust M-estimation framework for the BEKK model and proposed a modification – bounded innovation propagation – that limits the impact of extreme innovations on future volatility. Through both Monte Carlo simulations and an empirical application involving the Nasdaq and NYSE Composite indices, they demonstrated the strong robustness of the M-estimator under a fat-tailed Student's *t*-loss function. Their approach also improved minimum-variance portfolio allocations. However, the method has an important limitation: it restricts the impact of shocks on conditional variance according to the extremeness of the entire lagged return vector rather than that of individual returns. Consequently, the volatility of assets less affected by outliers may be underestimated.

[Croux et al. \(2010\)](#) proposed a robust version of the multivariate exponential smoothing model. Their simulation study showed that the robust method produced more accurate covariance forecasts in the presence of outliers and reduced the downward bias in the estimation of smoothing parameters. When applied to monthly housing data, the approach successfully generated cleaned series free from extreme observations.

[LaBarr \(2010\)](#) developed a robust estimation method for the DCC model based on a bounded deviance function. The approach yielded improved volatility estimates in both simulated data and foreign exchange series involving the U.S. dollar against major currencies. However, the estimation of correlation parameters remained comparatively less accurate.

[Boudt et al. \(2013\)](#) extended this line of research by introducing a robust DCC model and applying it to the EUR/USD and JPY/USD exchange rates. Their results demonstrated more accurate out-of-sample covariance forecasts than the standard DCC specification, as well as improved portfolio performance characterized by similar returns but lower turnover. Because this model forms part of our study, a detailed methodological description is provided in [Section 3.2](#).

Further contributions were made by [Iqbal \(2013a, 2013b\)](#), who proposed robust estimation methods for the simplified multivariate GARCH (S-GARCH; [Harris et al., 2007](#)) and orthogonal GARCH (O-GARCH; [Alexander & Chibumba, 1996](#)) models. Because these frameworks require only univariate GARCH estimation, they are computationally more tractable. Monte Carlo simulations revealed that estimators such as least absolute deviation (LAD) and the B-estimator outperformed quasi-maximum likelihood estimation (QMLE), particularly in forecasting correlations and VaR. Applications to equity and exchange rate data further confirmed their predictive advantages.

[Ebrahimi and Seyedhosseini \(2015\)](#) introduced a robust M-estimator for the multivariate FIGARCH model in BEKK form that incorporates bounded innovation effects. Their model was applied to the analysis of volatility transmission between WTI crude oil and the Tehran and Dubai stock indices.

The second promising strand of research concerns range-based volatility modeling. Comprehensive reviews are provided by [Chou et al. \(2015\)](#) and [Petropoulos et al. \(2022\)](#); therefore, we briefly summarize five recent contributions.

[De Nard et al. \(2022\)](#) proposed the IDR-DCC-NL model, which replaces close-to-close returns with intraday-regularized (OHLC-based) returns and employs range-based estimators as volatility proxies.

Applied to U.S. equities, the model improved global minimum-variance portfolio allocation and outperformed both the standard DCC and DCC-NL models.

[Fiszeder et al. \(2023a\)](#) introduced the DCC-OHLC model, which incorporates OHLC information into both the variance and correlation equations. Using [Molnár's \(2016\)](#) RGARCH model for conditional variances and [Popov's \(2016\)](#) OHLC-based correlation estimator, they demonstrated superior covariance forecasts and VaR predictions across ETF and foreign exchange datasets relative to standard and alternative range-based DCC specifications.

Building on this work, [Fiszeder et al. \(2023b\)](#) proposed the DCC-X-RGARCH-X model, an extension of the DCC-RGARCH framework that incorporates exogenous variables into the mean, variance, and covariance equations. Applied to ETFs representing oil, equities, and gold, the model outperformed competing specifications, with Google search intensity for the term “oil prices” serving as a significant predictor of covariances.

[Wu et al. \(2023\)](#) developed the bivariate VS-ACARR model, extending [Chou's \(2005\)](#) univariate CARR framework to capture volatility spillovers. Applied to WTI crude oil and Bitcoin, the model identified strong spillover effects from oil to Bitcoin and a weak leverage effect in Bitcoin, while delivering more accurate volatility forecasts than GARCH- and CARR-based alternatives.

[Yan et al. \(2025\)](#) introduced three new multivariate models: the DCC-REGARCH, co-range REGARCH (CRREGARCH), and co-range CARR (CRCARR). The DCC-REGARCH and CRREGARCH models exhibited the strongest out-of-sample performance in forecasting variance-covariance matrices for the GBP/USD and EUR/USD exchange rates, as well as the SPDR S&P 500 ETF Trust and the United States Oil Fund ETF. These models also generated the lowest portfolio variance and turnover for global minimum-variance portfolios in the majority of cases.

Despite these advances, a notable gap remains in the literature. Existing studies on robust multivariate volatility models focus primarily on handling outliers within return-based GARCH frameworks, whereas the literature on range-based volatility models emphasizes the informational advantages of price-range measures but typically does not address robustness to extreme observations.

The present study bridges these two strands of research by proposing multivariate volatility models that simultaneously incorporate robust estimation techniques and range-based volatility measures. In doing so, the paper extends both the literature on robust multivariate GARCH models and the literature on range-based volatility modeling.

3. Theoretical background

In this section, we first present two models from the literature that serve as the foundation for our proposed specifications: the cDCC model of [Aielli \(2013\)](#) ([Section 3.1](#)) and the BIP-cDCC model of [Boudt et al. \(2013\)](#) ([Section 3.2](#)). Combined with the DCC-RGARCH model of [Fiszeder et al. \(2019\)](#), these frameworks form the basis of the proposed BIP-cDCC-BIP-RGARCH and BIP-cDCC-BM-RGARCH models, which are described in [Sections 3.3 and 3.4](#), respectively.

3.1. The cDCC model of [Aielli \(2013\)](#)

In this subsection, we describe the standard cDCC model based on closing prices and its estimation using the quasi-maximum likelihood method. To clearly distinguish this specification from our proposed models, which rely on different univariate volatility equations, we refer to it as the cDCC-GARCH model. This parameterization provides a more tractable version of [Engle's \(2002\)](#) DCC model.

Let ϵ_t denote the $N \times 1$ vector of demeaned observed returns given by:

$$\epsilon_t | \Psi_{t-1} \sim N(0, \text{cov}_t) \quad (1)$$

where ψ_{t-1} denotes the information set available at time $t-1$, N denotes the multivariate normal distribution, and $\mathbf{cov}_t$ is the $N \times N$ symmetric conditional covariance matrix.

The cDCC(P, Q)-GARCH(p, q) model by Aielli (2013) can be written as:

$$\mathbf{cov}_t = \mathbf{D}_t \mathbf{cor}_t \mathbf{D}_t \quad (2)$$

$$\mathbf{cor}_t = \mathbf{Q}_t^{*-1} \mathbf{Q}_t \mathbf{Q}_t^{*-1} \quad (3)$$

$$\mathbf{Q}_t = \left(1 - \sum_{i=1}^Q \zeta_i - \sum_{j=1}^P \theta_j \right) \mathbf{S} + \sum_{i=1}^Q \zeta_i (\mathbf{Q}_{t-i}^* \mathbf{z}_{t-i} \mathbf{z}_{t-i}' \mathbf{Q}_{t-i}^*) + \sum_{j=1}^P \theta_j \mathbf{Q}_{t-j} \quad (4)$$

where $\mathbf{D}_t = \text{diag}(h_{1t}^2, h_{2t}^2, \dots, h_{Nt}^2)$, and the conditional variances h_{nt} (for $n = 1, 2, \dots, N$) are described by univariate GARCH models (Eqs. (5)–(6)). Furthermore, $\mathbf{z}_t$ denotes the standardized $N \times 1$ residual vector, assumed to be serially independent and given by $\mathbf{z}_t = \mathbf{D}_t^{-1} \epsilon_t$, $\mathbf{cor}_t$ is the time varying $N \times N$ conditional correlation matrix of $\mathbf{z}_t$, $\mathbf{S}$ is the unconditional $N \times N$ covariance matrix of $\mathbf{Q}_t^* \mathbf{z}_t$ and $\mathbf{Q}_t^*$ is the diagonal $N \times N$ matrix composed of the square roots of the diagonal elements of $\mathbf{Q}_t$. The parameters ζ_i (for $i = 1, 2, \dots, Q$), θ_j (for $j = 1, 2, \dots, P$) are nonnegative and satisfy the condition $\sum_{i=1}^Q \zeta_i + \sum_{j=1}^P \theta_j < 1$.

The univariate GARCH(p, q) model of Bollerslev (1986), used to calculate the conditional variances, is given by:

$$\epsilon_{nt} | \psi_{t-1} \sim N(0, h_{nt}), \quad n = 1, 2, \dots, N \quad (5)$$

$$h_{nt} = \alpha_{n0} + \sum_{i=1}^q \alpha_{ni} \epsilon_{n,t-i}^2 + \sum_{j=1}^p \beta_{nj} h_{n,t-j} \quad (6)$$

where $\epsilon_{nt} = r_{nt} - \gamma_{n0}$, r_{nt} denotes the return, and γ_{n0} is the constant. In most applications, it is assumed that $\alpha_{n0} > 0$, $\alpha_{ni} \geq 0$, and $\beta_{nj} \geq 0$ (for $n = 1, 2, \dots, N$; $i = 1, 2, \dots, q$; $j = 1, 2, \dots, p$). However, weaker conditions ensuring the non-negativity of the conditional variance may also be considered (Nelson & Cao, 1992). The covariance-stationarity condition for ϵ_{nt} is $\sum_{i=1}^q \alpha_{ni} + \sum_{j=1}^p \beta_{nj} < 1$.

The parameters of the cDCC model can be estimated using a two-stage quasi-maximum likelihood procedure. Let the parameter vector of the model, denoted by Θ , be partitioned into two groups such that $\Theta' = (\Theta_1, \Theta_2)$, where Θ_1 contains the parameters of the conditional mean and variance equations and Θ_2 contains the parameters of the correlation component. The log-likelihood function can then be expressed as the sum of two components:

$$L(\Theta) = L_{Vol}(\Theta_1) + L_{Corr}(\Theta_2 | \Theta_1) \quad (7)$$

where $L_{Vol}(\Theta_1)$ can be written as the sum of the log-likelihood functions of N univariate GARCH models:

$$L_{Vol}(\Theta_1) = -\frac{1}{2} \sum_{n=1}^N \left(T \ln(2\pi) + \sum_{t=1}^T \left(\ln(h_{nt}) + \frac{\epsilon_{nt}^2}{h_{nt}} \right) \right) \quad (8)$$

while $L_{Corr}(\Theta_2 | \Theta_1)$ represents the correlation component:

$$L_{Corr}(\Theta_2 | \Theta_1) = -\frac{1}{2} \sum_{t=1}^T (\ln |\mathbf{cor}_t| + \mathbf{z}_t' \mathbf{cor}_t^{-1} \mathbf{z}_t - \mathbf{z}_t' \mathbf{z}_t) \quad (9)$$

Accordingly, in the first stage, the parameters of the univariate GARCH models are estimated separately for each series. In the second stage, the parameters Θ_2 are estimated conditional on the parameter estimates $\hat{\Theta}_1$ obtained in the first stage. The estimation of the matrix $\mathbf{S}$ and the parameters Θ_2 is interdependent because $\mathbf{S}$ is updated sequentially as the correlation matrix of $\mathbf{Q}_t^* \mathbf{z}_t$. In contrast, in Engle's (2002) DCC model, the matrix $\mathbf{S}$ is computed only once prior to the estimation of the parameters Θ_2 .

3.2. The BIP-cDCC model of Boudt et al. (2013)

Boudt et al. (2013) introduced a robust dynamic conditional correlation model and proposed three modifications to the original cDCC framework. First, the standard GARCH specification is replaced with the BIP-GARCH model. Second, the impact of large innovations on the correlation matrix is bounded through the BIP procedure applied in the updating equation for $\mathbf{Q}_t$. Third, a robust procedure is employed to estimate the unconditional correlation matrix. To clearly distinguish this specification from related models, we refer to it as the BIP-cDCC-BIP-GARCH model.

The BIP-cDCC(P, Q)-BIP-GARCH(p, q) model can be expressed as:

$$\epsilon_t | \psi_{t-1} \sim t_\nu(0, \mathbf{cov}_t) \quad (10)$$

$$\mathbf{cov}_t = \mathbf{D}_t \mathbf{cor}_t \mathbf{D}_t \quad (11)$$

$$\mathbf{cor}_t = \mathbf{Q}_t^{*-1} \mathbf{Q}_t \mathbf{Q}_t^{*-1} \quad (12)$$

$$\mathbf{Q}_t = \left(1 - \sum_{i=1}^Q \zeta_i - \sum_{j=1}^P \theta_j \right) \mathbf{S}_{RW} + \sum_{i=1}^Q \zeta_i \omega_{k_{\delta,N}}(d_{t-1}) (\mathbf{Q}_{t-i}^* \mathbf{z}_{t-i} \mathbf{z}_{t-i}' \mathbf{Q}_{t-i}^*) + \sum_{j=1}^P \theta_j \mathbf{Q}_{t-j} \quad (13)$$

where t_ν denotes the multivariate Student's t -distribution with ν degrees of freedom, $d_{t-1} = \mathbf{z}_{t-1}' \mathbf{cor}_{t-1}^{-1} \mathbf{z}_{t-1}$ is the squared Mahalanobis distance,

$\omega_{k_{\delta,N}}(u) = c_{\delta,N} \min\left(1, \frac{k_{\delta,N}}{u}\right)$ is a weighting function, $c_{\delta,N} =$

$\frac{1}{F_{\chi^2_{N+2}}(\chi^2_N(\delta) + (1-\delta)\chi^2_N(\delta))}$ is a correction factor, and $k_{\delta,N}$ is the δ quantile of the

chi-square distribution with N degrees of freedom. The introduction of the correction factor $c_{\delta,N}$ follows from the use of the threshold-based weighting function $\omega_{k_{\delta,N}}$. Specifically, the factor rescales the weighted term so that, after the filtering implied by the weighting function, the conditional expectation of the weighted term remains equal to the conditional correlation in the absence of outliers. The remaining variables are defined analogously to those in the cDCC-GARCH model. The choice of δ reflects the standard efficiency-robustness trade-off. In the proposed models k_δ is determined empirically (details are provided in Sections 3.3 and 3.4).

The sample correlation matrix $\mathbf{S}$ model may be biased in the presence of additive jumps. To address this issue, Boudt et al. (2013) employ the reweighted unconditional correlation matrix estimator:

$$\mathbf{S}_{RW} = \mathbf{RC}^{*-1} \mathbf{RC} \mathbf{RC}^{*-1} \quad (14)$$

where $\mathbf{RC}^*$ is the diagonal $N \times N$ matrix containing the square roots of the diagonal elements of $\mathbf{RC}$, while $\mathbf{RC}$ is defined as $\mathbf{RC} =$

$\frac{c_{0.95,N}}{\sum_{t=1}^T L_t} \sum_{t=1}^T \mathbf{z}_t \mathbf{z}_t' L_t$, with weights $L_t = I[\mathbf{z}_t' \mathbf{S} \mathbf{C}_t^{-1} \mathbf{z}_t \leq \chi^2_N(0.95)]$. The cor-

rected Spearman correlation matrix is given by $\mathbf{S} \mathbf{C}_t = 2 \sin\left(\frac{1}{6} \pi \mathbf{C}_t\right)$.

The Spearman correlation matrix $\mathbf{C}_t$ is computed within a local window centered around each observation $\mathbf{z}_t$. This window spans the interval $\left[t - \frac{K}{2}, t + \frac{K}{2}\right]$. Near the sample boundaries, when $t < K/2$, the interval is defined as $[1, K+1]$, whereas when $t > n - K/2$, it is defined as $[T-K, T]$. Boudt et al. (2013) set $K = 30$.

Conditional variances are modeled using the BIP-GARCH specification, which takes the following form:

$$\epsilon_{nt} | \psi_{t-1} \sim t_\nu(0, h_{nt}), \quad n = 1, 2, \dots, N \quad (15)$$

$$h_{nt} = \alpha_{n0} + \sum_{i=1}^q \alpha_{ni} \omega_{k_{\delta,1}} \left(\frac{c_{n,t-i}^2}{h_{n,t-i}} \right) e_{n,t-i}^2 + \sum_{j=1}^p \beta_{nj} h_{n,t-j} \quad (16)$$

The unconditional mean return is estimated using a robust reweighted sample mean:

$$\hat{\gamma}_{n0} = \frac{\sum_{t=1}^T r_{nt} I_{nt}}{\sum_{t=1}^T I_{nt}}, \quad n = 1, 2, \dots, N \quad (17)$$

where $I_{nt} = I \left[\frac{(r_{nt} - \text{median}_i(r_{nt}))^2}{\text{MAD}_t^2} \leq l \right]$, I is an indicator function, MAD_t denotes the median absolute deviation $m \cdot \text{median}_i(|r_{nt} - \text{median}_i(r_{nt})|)$, with the correction factor m set to 1.483, and l is the threshold corresponding to the 0.95 quantile of the chi-square distribution. The correction factor m is set equal to 1.483 to ensure that MAD_t , when used as a volatility estimator, is consistent with the corresponding volatility measure under the assumption of normality. Both the local median_i and MAD_t are computed within moving windows analogous to those used in the estimation of the Spearman correlation matrix.

The parameters of the BIP-cDCC-GARCH model are estimated using an M-estimation procedure implemented in two stages. Let the parameter vector Θ be partitioned into two components $\Theta' = (\Theta_1, \Theta_2)$, where Θ_1 contains the parameters governing the conditional variances and Θ_2 contains the parameters of the correlation component. In the first stage, the parameters of the univariate BIP-GARCH models are estimated separately for each series by minimizing the average value of the objective function $\rho(\cdot)$, evaluated at the logarithm of the squared devolatilized residuals:

$$\hat{\Theta}_1 = \arg \min_{\Theta_1} \sum_{n=1}^N \left[\frac{1}{T} \sum_{t=1}^T \rho \left(\ln \frac{e_{nt}^2}{h_{nt}} \right) \right] \quad (18)$$

where $\rho(\cdot)$ is associated with the Student's t_4 density function, i.e., $\rho(z) = -z + \sigma_{1,4} \rho_{t_{1,4}}(\exp(z))$, $\sigma_{N,\nu} = \frac{N}{E[\rho'_{N,\nu}(u)u]}$, $\rho_{t_{N,\nu}}(u) = (N + \nu) \ln \left(1 + \frac{u}{\nu-2} \right)$.

In the second stage the parameters Θ_2 are estimated conditional on the first-stage estimates $\hat{\Theta}_1$. The M-estimator for the correlation component is given by:

$$\hat{\Theta}_2 = \arg \min_{\Theta_2} \sum_{t=1}^n [\ln |\text{cor}_t| + \sigma \rho(\mathbf{z}_t' \text{cor}_t^{-1} \mathbf{z}_t)] \quad (19)$$

where σ is a correction factor equal to $\sigma_{N,4}$, $\rho(u) = \rho_{t_{N,4}}(u)$.

Note that when $\rho(z) = z$ and $\sigma = 1$, the Gaussian QML estimator is obtained as a special case.

The matrix $\mathbf{S}_{RW}$ and the parameter vector Θ_2 are estimated iteratively. The procedure begins with the estimation of $\mathbf{S}_{RW}$ as the reweighted unconditional correlation matrix of $\mathbf{Q}_1^* \mathbf{z}_1, \dots, \mathbf{Q}_T^* \mathbf{z}_T$. Subsequently, the algorithm alternates between updating $\mathbf{S}_{RW}$ and estimating Θ_2 using Student t_4 M-estimation within the BIP-cDCC framework until convergence is achieved.

3.3. The BIP-cDCC-BIP-RGARCH model

In this subsection, we introduce the first of the proposed models, referred to as the BIP-cDCC-BIP-RGARCH model. This specification combines the BIP-cDCC-BIP-GARCH model of Boudt et al. (2013) with the DCC-RGARCH model of Fiszeder et al. (2019). To ensure that the BIP procedure is appropriate for both the RGARCH framework and the analyzed time series, we introduce several modifications.

The BIP-cDCC(P, Q)-BIP-RGARCH(p, q) model can be written as:

$$\epsilon_t | \psi_{t-1} \sim D(0, \text{cov}_t) \quad (20)$$

$$\text{cov}_t = \mathbf{D}_t \text{cor}_t \mathbf{D}_t \quad (21)$$

$$\text{cor}_t = \mathbf{Q}_t^{*-1} \mathbf{Q}_t \mathbf{Q}_t^{*-1} \quad (22)$$

$$\mathbf{Q}_t = \left(1 - \sum_{i=1}^Q \zeta_i - \sum_{j=1}^P \theta_j \right) \mathbf{S}_{RW} + \sum_{i=1}^Q \zeta_i \omega_{k_{\delta,N}}(d_{t-1}) (\mathbf{Q}_{t-i}^* \mathbf{z}_{t-i} \mathbf{z}_{t-i}' \mathbf{Q}_{t-i}^*) + \sum_{j=1}^P \theta_j \mathbf{Q}_{t-j} \quad (23)$$

where D denotes either the multivariate normal distribution or the multivariate Student's t -distribution. The constant parameters of the weighting function $\omega_{k_{\delta,N}}(u) = c_{\delta,N} \min \left(1, \frac{k_{\delta,N}}{u} \right)$, namely the scaling factor $c_{\delta,N}$ and the threshold $k_{\delta,N}$ are calibrated using the empirical distributions of the observed time series.¹ Specifically, the values of $k_{\delta,N}$ are determined as quantiles of the empirical distribution of $\mathbf{z}_t' \text{cor}_t^{-1} \mathbf{z}_t$ while the constants $c_{\delta,N}$ are chosen so that the weighting scheme preserves the conditional expectation of the weighted term under the absence of outliers, thereby ensuring consistency with the conditional correlation structure.

To determine these constants, the empirical distribution of $\mathbf{z}_t' \text{cor}_t^{-1} \mathbf{z}_t$ is obtained from a preliminary estimation based on the standard DCC- n model. This procedure replaces the constants derived from the chi-square distribution, which rely on the restrictive and often unrealistic assumption of normally distributed returns. The modification is introduced to ensure consistency between the multivariate and univariate components of the model. In the univariate specification, theoretically derived constants cannot be employed because the standard variance estimator is replaced with a more efficient volatility estimator, as discussed in detail below in the description of the univariate component. The estimated values of these constants are reported in Tables 1 and 2. The remaining variables and parameters are defined analogously to those in the BIP-cDCC-BIP-GARCH model. The choice of the filtering level δ reflects the standard efficiency-robustness trade-off. In this study, we consider four values of δ , hereafter referred to as filtration levels: 0.99, 0.975, 0.95, and 0.90.

Conditional variances are modeled using the BIP-RGARCH specification:

$$\epsilon_{nt} | \psi_{t-1} \sim d(0, h_{nt}), \quad n = 1, 2, \dots, N \quad (24)$$

$$h_{nt} = \alpha_{n0} + \sum_{i=1}^q \alpha_{ni} \omega_{k_{\delta,1}} \left(\frac{\sigma_{GK,n,t-i}^2}{h_{n,t-i}} \right) \sigma_{GK,n,t-i}^2 + \sum_{j=1}^p \beta_{nj} h_{n,t-j} \quad (25)$$

where d denotes the univariate normal or Student's t -distribution, $\sigma_{GK,n,t}^2 = 0.5 [\ln(H_{nt}/L_{nt})]^2 - (2\ln 2 - 1) [\ln(C_{nt}/O_{nt})]^2$, represents the Garman-Klass variance estimator (Garman & Klass, 1980). The variables O_{nt} , H_{nt} , L_{nt} , C_{nt} denote the daily open, high, low, and close prices, respectively, of the n -th asset. Although the Garman-Klass estimator is employed in the baseline specification, alternative range-based variance estimators can also be used. The weighting function $\omega_{k_{\delta,1}}$ is adjusted in the same manner as in the $\mathbf{Q}_t$ equation. Specifically, the constants $c_{\delta,1}$ and $k_{\delta,1}$ are calibrated to the empirical distributions of the observed time series rather than being derived under the chi-square assumption. This adjustment is necessary because the standard squared-return term in the variance equation is replaced by an efficient range-based variance estimator. Consequently, the correction factor derived from the theoretical distribution of squared returns is no longer appropriate. Instead,

¹ The parameter adjustments are done with the use of the empirical data from the first rolling window and rely on preliminary estimates obtained from a basic DCC- n model.

Table 1Adjusted values of the threshold parameter $k_{\delta,N}$ based on the empirical distribution of returns.

Filtering level δ	Models with the normal distribution				Models with the Student's t-distribution			
	$N = 1$	$N = 2$	$N = 3$	$N = 4$	$N = 1$	$N = 2$	$N = 3$	$N = 4$
0.9	2.415	4.609	6.670	8.585	2.343	4.556	6.619	8.358
0.95	4.014	7.179	9.738	12.582	4.005	7.046	9.870	12.375
0.975	6.212	10.277	13.568	16.361	6.046	10.320	13.409	16.454
0.99	10.144	14.850	19.271	22.938	10.161	15.000	19.817	24.663

Note: The table presents adjusted values of the $k_{\delta,N}$ parameter of the weight function $w_{k_{\delta,N}}$ based on estimates obtained from the empirical time series.**Table 2**Adjusted values of the correction factor $c_{\delta,N}$ based on the empirical distribution of returns.

Filtering level δ	Models with the normal distribution				Models with the Student's t-distribution			
	$N = 1$	$N = 2$	$N = 3$	$N = 4$	$N = 1$	$N = 2$	$N = 3$	$N = 4$
0.9	1.512	1.320	1.248	1.210	1.563	1.352	1.276	1.243
0.95	1.291	1.178	1.143	1.116	1.322	1.209	1.163	1.140
0.975	1.172	1.107	1.086	1.076	1.204	1.128	1.108	1.096
0.99	1.089	1.062	1.051	1.046	1.110	1.082	1.069	1.060

Note: Adjusted parameter values are obtained from empirical return distributions estimated from the sample data.

the parameters are calibrated to preserve the same distributional characteristics as those implied by the original BIP framework, while their numerical values are determined empirically from the sample rather than analytically. In this way, the conceptual interpretation of the parameters is retained while adapting them to the distributional properties of the alternative variance estimators employed in the model. This approach follows the empirical correction procedure proposed by Fiszeder et al. (2024). The values of the correction factors adjusted for the modified variance Eq. (25) based on the Garman–Klass estimator are reported in Table 3.

The unconditional mean return, used to obtain the demeaned returns ϵ_{nt} is estimated using the robust reweighted estimator in Eq. (17). The constants l and m are adjusted, analogously to the previous parameter corrections, to match the sample characteristics of the empirical distributions. For the threshold l we use the quantile $k_{0.95,1}$ (the choice of the 0.95 quantile follows the recommendation of Boudt et al., 2013), whereas for the correction factor m we use a value 1.898. This value ensures the consistency of the MAD_t estimator with the volatility of the empirical time series in the absence of outliers.

The parameters of the BIP-cDCC-BIP-RGARCH model can be estimated in the same manner as those of the BIP-cDCC-BIP-GARCH model, with appropriately adjusted values of the $\sigma_{N,4}$ parameter, reported in

Table 3Adjusted values of the weighting-function parameters $k_{\delta,1}$ and $c_{\delta,1}$ for univariate models.

Filtering level δ	$k_{\delta,1}$		$c_{\delta,1}$	
	Normal	Student's t_ν	Normal	Student's t_ν
0.9	1.992	1.939	1.247	1.257
0.95	2.942	2.798	1.152	1.167
0.975	4.172	4.030	1.097	1.110
0.99	6.350	6.489	1.059	1.066

Notes: The reported values correspond to the adjusted weighting-function parameters used with the Garman–Klass volatility estimator. The adjustments are based on empirical distributions estimated from the sample data.

Table 4Adjusted values of the correction factor $\sigma_{N,4}$ for Student's t-distribution models.

$N = 1$	$N = 2$	$N = 3$	$N = 4$
0.974	0.949	0.936	0.929

Note: The table reports adjusted values of the correction factor $\sigma_{N,4}$, originally proposed by Boudt et al. (2013), obtained from empirical time series.

Table 4. However, when the normal distribution is selected in specification (20), the function $\rho(z)$ must be modified accordingly, i.e.:

$$\rho(z) = -\log\left((2\pi)^{-0.5} e^{-0.5(\epsilon^2 - z)}\right) \quad (26)$$

In addition, both $\rho(z)$ and σ must correspond to the Gaussian case, implying that $\rho(z) = z$ and $\sigma = 1$.

3.4. The BIP-cDCC-BM-RGARCH model

Our second proposed robust multivariate range-based model is presented in this subsection and will be referred to as the BIP-cDCC-BM-RGARCH model.² The covariance component of the model is similar to that of the BIP-cDCC-BIP-RGARCH specification; however, the conditional variances are modeled using the BM-RGARCH model of Fiszeder et al. (2024) rather than the BIP-RGARCH model.

The BIP-cDCC(P, Q)-BM-RGARCH(p, q) model can be written as:

$$\epsilon_t | \psi_{t-1} \sim D(0, \text{cov}_t) \quad (27)$$

$$\text{cov}_t = \mathbf{D}_t \text{cor}_t \mathbf{D}_t \quad (28)$$

$$\text{cor}_t = \mathbf{Q}_t^{*-1} \mathbf{Q}_t \mathbf{Q}_t^{*-1} \quad (29)$$

² The acronym BM follows Muler and Yohai (2008) and refers to: B – “bounding” the influence of outliers in past volatility (evaluated as squared residuals) on subsequent volatility estimates through the filtering function, and M – the M-estimator employed instead of the standard ML estimator.

$$\mathbf{Q}_t = \left(1 - \sum_{i=1}^Q \zeta_i - \sum_{j=1}^P \theta_j\right) \mathbf{S}_{RW} + \sum_{i=1}^Q \zeta_i \omega_{k_{\delta,N}}(\mathbf{d}_{t-1}) (\mathbf{Q}_{t-i}^* \mathbf{z}_{t-i} \mathbf{z}_{t-i}' \mathbf{Q}_{t-i}^*) + \sum_{j=1}^P \theta_j \mathbf{Q}_{t-j} \quad (30)$$

with the variables and parameters defined analogously to those in the BIP-cDCC(P, Q)-BIP-RGARCH(p, q) model.

The univariate BM-RGARCH(p, q) model used to calculate conditional variances is given by:

$$\epsilon_{nt} | \psi_{t-1} \sim d(0, h_{nt}^*), \quad n = 1, 2, \dots, N \quad (31)$$

$$h_{nt}^* = \alpha_{n0} + \sum_{i=1}^q \alpha_{ni} \omega_{k_{\delta,1}}^* \left(\frac{\sigma_{GK,n}^2 t-i}{h_{nt}^* t-i} \right) \sigma_{GK,n}^2 t-i + \sum_{j=1}^p \beta_{nj} h_{nt-j}^* \quad (32)$$

where the weighting function does not include the correction factor, i.e.

$\omega_{k_{\delta,1}}^*(u) = \min\left(1, \frac{k_{\delta,1}}{u}\right)$, and the threshold $k_{\delta,N}$ is the empirical δ quantile calibrated to the observed data.

The M-estimator is obtained by minimizing and comparing the following objective functions:

$$\hat{\varsigma}_n = \begin{cases} \hat{\varsigma}_{n1} & \text{if } M(\hat{\varsigma}_{n1}) \leq M^*(\hat{\varsigma}_{n2}) \\ \hat{\varsigma}_{n2} & \text{if } M(\hat{\varsigma}_{n1}) > M^*(\hat{\varsigma}_{n2}) \end{cases} \quad (33)$$

where $\hat{\varsigma}_{n1} = \arg \min_{\varsigma_n \in C} M(\varsigma_n)$, $\hat{\varsigma}_{n2} = \arg \min_{\varsigma_n \in C} M^*(\varsigma_n)$ for some convenient compact set C , functions M and M^* have the forms:

$$M(\varsigma_n) = -\left(\frac{T}{2}\right) \ln(2\pi) + 1/T \sum_{t=1}^T \rho\left(\ln \frac{\epsilon_{nt}^2}{h_{nt}(\varsigma_n)}\right) \quad (34)$$

$$M^*(\varsigma_n) = -\left(\frac{T}{2}\right) \ln(2\pi) + 1/T \sum_{t=1}^T \rho\left(\ln \frac{\epsilon_{nt}^2}{h_{nt}^*(\varsigma_n)}\right) \quad (35)$$

where function ρ is defined as $\rho(z) = -\log\left((2\pi)^{-0.5} e^{-0.5(e^z - z)}\right)$ for the Gaussian case and $\rho(z) = -z + \sigma_{1,\nu} \rho_{t_{1,\nu}}(\exp(z))$ for the Student's t-distribution with ν degrees of freedom. Furthermore, h_{nt} denotes the variance equation corresponding to Eq. (32) with the weighting function omitted.

The parameters of the covariance component of the BIP-cDCC-BM-RGARCH model can be estimated in the same manner as the covariance parameters of the BIP-cDCC-BIP-RGARCH model.

4. Description of data

We analyze four markets: oil, gold, equities, and Bitcoin. To this end, we use three exchange-traded funds (ETFs) listed on the NYSE Arca, together with the BTC/USD cryptocurrency pair. The ETFs include the United States Oil Fund (USO), which primarily holds short-term NYMEX WTI crude oil futures and other oil-related instruments; SPDR Gold Shares (GLD), which holds physical gold bullion; and SPDR Portfolio S&P 500 Growth (SPYG), which tracks large-cap growth stocks included in the S&P 500 index. Throughout the paper, we refer to these instruments as oil, gold, and equities, respectively. For Bitcoin, the data are obtained directly from the Kraken cryptocurrency exchange.

Our objective is to generate more accurate volatility and covariance forecasts using only daily data. Accordingly, daily observations are employed both for model estimation and forecasting, whereas intraday

data are used exclusively for forecast evaluation. The sample period spans from July 1, 2016, to June 28, 2024.³

Overnight volatility and opening jumps may introduce additional noise into return series. To mitigate this issue, we analyze open-to-close returns rather than close-to-close returns. Specifically, we use percentage logarithmic returns calculated as $r_{nt} = 100 \ln(C_{nt}/O_{nt})$, where C_{nt} and O_{nt} denote the closing and opening prices of the n -th asset at time t , respectively. Open-to-close returns are employed to avoid the noise associated with overnight volatility; however, the main findings of our study are also robust to the use of close-to-close returns. To calculate the range-based variance estimators, we use daily open, high, low, and close prices. As a proxy for daily variance, we employ realized variance computed as the sum of squared 15-min returns. Similarly, realized covariances, calculated as the sum of the cross-products of 15-min returns, are used as proxies for daily covariance.

An important issue in empirical studies that combine traditional financial assets and cryptocurrencies concerns the synchronization of trading hours. Exchange-traded funds and other exchange-traded assets are traded during fixed exchange sessions, whereas cryptocurrencies such as Bitcoin trade continuously on a 24-h basis. To ensure comparability across markets and avoid nonsynchronous trading effects, all daily range-based measures in this study are constructed using a common trading window corresponding to U.S. exchange trading hours. Specifically, the daily open, high, low, and close prices are calculated over the interval from 13:30 to 20:00 UTC during the daylight-saving period and from 14:30 to 21:00 UTC during the standard-time period, corresponding to the regular trading hours of U.S. exchanges. For Bitcoin, the same time window is used when computing daily OHLC prices. This synchronization ensures that the volatility and covariance measures are based on aligned trading intervals across all assets. The same trading window is also used when constructing realized -volatility proxies from intraday data.

Fig. 1 presents the time series of prices, returns, and volatilities for all analyzed assets. Descriptive statistics for returns and realized variances are reported in Table 5. The summary statistics indicate that average returns are positive for Bitcoin and equities but negative for oil and gold. Bitcoin is the most profitable investment, however, it is also associated with the highest level of risk. The standard deviation of Bitcoin returns is nearly 2.5 times higher than that of oil, almost five times higher than that of equities, and more than seven times higher than that of gold. The return distributions exhibit negative skewness, whereas the distributions of realized variances are positively skewed. All return and realized-variance distributions display high kurtosis, which is particularly pronounced for realized variances.

Large outliers in both returns and realized variances are clearly visible in Fig. 1, particularly for Bitcoin and oil. Some of the most extreme observations occurred during the outbreak of the COVID-19 pandemic in March 2020. These patterns provide strong support for the use of robust estimation methods.

5. Parameter estimates in multivariate volatility models

We consider six DCC-type models:

- 1) The DCC-GARCH model of Engle (2002).

³ The data is available at the following address: <https://doi.org/10.18150/AN5NET>.



Fig. 1. Daily prices, returns, and realized variances of oil, gold, equities, and Bitcoin.

2) The cDCC-GARCH model of [Aielli \(2013\)](#) (Section 3.1), which represents a more tractable version of the DCC-GARCH model.

3) The DCC-RGARCH model of [Fiszeder et al. \(2019\)](#). In this specification, the univariate GARCH component of the DCC model is replaced by the RGARCH model.

4) The BIP-cDCC-BIP-GARCH model of [Boudt et al. \(2013\)](#) (Section 3.2), which constitutes a robust version of the cDCC-GARCH model.

5) The first proposed model, BIP-cDCC-BIP-RGARCH (Section 3.3). In this specification, the univariate BIP-GARCH model within the BIP-cDCC-BIP-GARCH framework is replaced by the BIP-RGARCH model.

6) The second proposed model, BIP-cDCC-BM-RGARCH (Section

3.4). In this specification, the univariate BIP-GARCH model within the BIP-cDCC-BIP-GARCH framework is replaced by the BM-RGARCH model.

To evaluate the performance of the proposed models, we compare them with several benchmark specifications representing well-established approaches in the multivariate volatility literature. The cDCC-GARCH model serves as a standard benchmark in empirical applications because of its widespread use and improved statistical properties relative to the original DCC specification. The BIP-cDCC-BIP-GARCH model introduces robustness to outliers through bounded innovation propagation. Finally, the DCC-RGARCH model incorporates

Table 5

Descriptive statistics of daily returns and realized variances for ETFs and BTC.

Asset	Mean	Min	Max	Standard dev.	Skewness	Kurtosis
Returns						
Oil	−0.055	−14.842	11.616	1.642	−0.724	8.591
Gold	−0.006	−4.154	3.149	0.574	−0.185	4.47
Equities	0.016	−5.777	4.682	0.843	−0.273	4.911
Bitcoin	0.157	−48.674	22.47	4.093	−0.931	12.553
Realized volatility						
Oil	3.196	0.135	462.429	13.732	25.556	765.638
Gold	0.325	0.02	8.371	0.497	7.85	88.537
Equities	0.708	0.005	32.495	1.922	11.053	149.319
Bitcoin	5.677	0.065	227.035	12.765	8.467	104.148

Note: The sample period spans from July 1, 2016, to June 28, 2024.

range-based volatility measures, which have been shown to provide more efficient volatility estimates than those based solely on closing prices. These models therefore constitute natural benchmarks for assessing whether the proposed robust multivariate range-based specifications improve both robustness and forecasting performance.

For the three variants of the robust models, we consider four filtration levels – 0.99, 0.975, 0.95, and 0.9 – resulting in twelve robust specifications. In addition, all models are estimated under two distributional assumptions: the normal distribution and the Student's *t*-distribution, the latter allowing for heavy-tailed return distributions. Combined with the standard DCC and cDCC models, the DCC-RGARCH specifications, and the robust variants, this results in a total of thirty models. For all RGARCH-based specifications, the Garman–Klass variance estimator is employed.

We begin by comparing the in-sample fit of the models. The parameters of all specifications are estimated using the full sample, spanning the period from July 1, 2016, to June 28, 2024. The GARCH, RGARCH, and DCC models are implemented with first-order lags, i.e., $p = 1$, $q = 1$ in Eqs. (6), (16), (25), and (32), and $P = 1$, $Q = 1$ in Eqs. (4), (13), (23), and (30).

The estimation results are reported in Table 6. For brevity, we present only the estimated coefficients of the correlation component, namely ζ_1 and θ_1 . The results for the univariate models are consistent with those reported by Fiszeder et al. (2024) and are available from the authors upon request.

Across all models, estimates of the ζ_1 parameter are lower and estimates of the θ_1 parameter are higher when the conditional Student's *t*-distribution is assumed than when the conditional normal distribution is used. In addition, the ζ_1 parameter tends to be slightly higher and the θ_1 parameter slightly lower in DCC-RGARCH models than in DCC-GARCH models, which is consistent with the findings of Fiszeder et al. (2023b). For positive covariances, a higher value of ζ_1 indicates that past shocks exert a stronger influence on current covariances, implying a faster response to changing market conditions. For negative covariances, however, this relationship may be reversed.

In most cases, the proposed robust DCC-RGARCH models yield higher ζ_1 estimates and lower θ_1 estimates than their standard counterparts. Similar patterns were reported by LaBarr (2010) for robust DCC-GARCH models applied to U.S. dollar exchange rates against the euro, British pound, Swiss franc, Japanese yen, and Canadian dollar. Likewise, Boudt et al. (2013), using the BIP-cDCC-BIP-GARCH model for S&P 500 sector indices, found higher ζ_1 estimates in the robust model than in the standard cDCC-GARCH specification for the energy, industrials, and information technology sectors, whereas the opposite pattern was observed for the remaining sectors.

The increase in the standard errors of the correlation parameters in the robust specifications is a theoretically expected consequence of the well-known robustness–efficiency trade-off. As formally demonstrated by Muler and Yohai (2008) for the BM-estimator and discussed by Boudt and Croux (2010) and Boudt et al. (2013) in the context of multivariate

GARCH models, robust estimators achieve resistance to outliers by down-weighting or truncating extreme observations, for example through the bounded innovation propagation mechanism. Although this filtering process successfully stabilizes the conditional covariance matrix and prevents the propagation of outliers, it inevitably reduces the effective amount of information available for parameter estimation. Consequently, the asymptotic variance of robust estimators is larger than that of standard QML estimators, resulting in larger standard errors. Therefore, the higher standard errors and lower in-sample statistical significance of ζ_1 and θ_1 can be viewed as a natural statistical premium paid for improved out-of-sample predictive robustness. As demonstrated by the out-of-sample results presented in the following sections, this trade-off proves highly beneficial, as the robust models substantially outperform the standard specifications in terms of both HMSE and Value-at-Risk forecasting accuracy.

Fig. 2 compares the time-varying conditional correlations estimated using two selected DCC specifications: the standard range-based DCC-RGARCH-*n* model and the robust range-based BIP-cDCC-BM-RGARCH-*n* model with a filtration level of 0.90, which achieves the best HMSE performance (see Section 6).

The weakest correlations are observed between oil and Bitcoin, fluctuating around zero, whereas the strongest occur between oil and equities, with an average value of approximately 0.23. Following the Russian invasion of Ukraine in 2022, the correlations between oil and equities, as well as between oil and Bitcoin, declined sharply and became negative. For the oil–equity pair, this decline resulted in the lowest correlations observed during the entire sample period. At the same time, oil–gold correlations increased substantially, reaching their highest values within the sample.

Gold and equities also exhibit noteworthy dynamics. Immediately after the outbreak of the war, their correlations fell below −0.2, representing the lowest values recorded in the dataset. Subsequently, however, the relationship strengthened, rising above 0.3 by the end of 2022, which represents the highest level observed during the sample period. The second-strongest correlations are found between equities and Bitcoin, averaging approximately 0.15 and increasing markedly during 2022.

Having identified differences between the conditional correlations generated by the robust range-based models and those produced by the benchmark specifications, we further examine the time-series properties of these discrepancies. As shown in Fig. A.1 in Appendix A, the correlation differences exhibit substantial persistence over time. Interestingly, the divergence between the models during periods of elevated volatility (e.g., the COVID-19 shock of 2020) is highly asset-pair specific. Although high volatility consistently leads to differences in correlation estimates, both the magnitude and direction of these differences vary across asset pairs, highlighting the complex nature of risk transmission captured by the robust range-based filtering mechanism (Fig. A.2 in Appendix A).

Because robustness to extreme observations is a key motivation for

Table 6

The results of parameter estimation for the correlation component.

Model	$\zeta_1(\text{SE}(\zeta_1))$	$\theta_1(\text{SE}(\theta_1))$
GARCH models		
DCC-n	0.019 (0.002)	0.954 (0.017)
DCC-t	0.014 (0.003)	0.971 (0.031)
cDCC-n	0.020 (0.002)	0.953 (0.016)
cDCC-t	0.016 (0.004)	0.966 (0.038)
RGARCH models		
DCC-n	0.020 (0.002)	0.951 (0.015)
DCC-t	0.017 (0.004)	0.962 (0.035)
Robust GARCH models, filtration 0.99		
BIP-cDCC-n	0.020 (0.016)	0.952 (0.116)
BIP-cDCC-t	0.016 (0.019)	0.966 (0.155)
Proposed models: robust RGARCH, filtration 0.99		
BIP-cDCC-n	0.021 (0.015)	0.949 (0.104)
BIP-cDCC-t	0.018 (0.018)	0.960 (0.135)
BIP*-cDCC-n	0.021 (0.016)	0.948 (0.112)
BIP*-cDCC-t	0.017 (0.018)	0.960 (0.126)
Robust GARCH models, filtration 0.975		
BIP-cDCC-n	0.020 (0.014)	0.955 (0.100)
BIP-cDCC-t	0.016 (0.017)	0.969 (0.123)
Proposed models: robust RGARCH, filtration 0.975		
BIP-cDCC-n	0.022 (0.014)	0.947 (0.095)
BIP-cDCC-t	0.016 (0.015)	0.965 (0.103)
BIP*-cDCC-n	0.021 (0.014)	0.948 (0.100)
BIP*-cDCC-t	0.017 (0.019)	0.959 (0.134)
Robust GARCH models, filtration 0.95		
BIP-cDCC-n	0.020 (0.015)	0.958 (0.105)
BIP-cDCC-t	0.017 (0.018)	0.965 (0.123)
Proposed models: robust RGARCH, filtration 0.95		
BIP-cDCC-n	0.022 (0.012)	0.951 (0.085)
BIP-cDCC-t	0.017 (0.013)	0.963 (0.093)
BIP*-cDCC-n	0.020 (0.015)	0.954 (0.110)
BIP*-cDCC-t	0.018 (0.018)	0.960 (0.133)
Robust GARCH models, filtration 0.9		
BIP-cDCC-n	0.021 (0.015)	0.954 (0.106)
BIP-cDCC-t	0.017 (0.018)	0.969 (0.121)
Proposed models: robust RGARCH, filtration 0.9		
BIP-cDCC-n	0.022 (0.016)	0.953 (0.117)
BIP-cDCC-t	0.017 (0.026)	0.963 (0.192)
BIP*-cDCC-n	0.022 (0.016)	0.946 (0.120)
BIP*-cDCC-t	0.018 (0.019)	0.960 (0.139)

Notes: For all RGARCH models, the Garman-Klass variance estimator is applied. BIP*-cDCC denotes the BIP-cDCC-BM-RGARCH model. Standard errors are reported in parentheses. The estimation sample spans July 1, 2016 – June 28, 2024.

the proposed models, we provide an additional descriptive illustration of their behavior during periods of large return shocks. Fig. 3 presents the relationship between Bitcoin return shocks and the differences in conditional correlations obtained from the standard and robust specifications for the Bitcoin-gold pair. The figure suggests that the differences between the models tend to increase during periods of large shocks, indicating that the robust filtering mechanism becomes particularly relevant in the presence of extreme observations.

6. Accuracy of covariance forecasts

We generate out-of-sample one-day-ahead forecasts of the

covariance matrix for each of the thirty models under consideration. Model parameters are estimated using a rolling-window approach with daily updates, and the window size is set to 1000 observations. The calibration of the filtering constants is based exclusively on information available within the initial in-sample estimation window. No observations from the out-of-sample evaluation period are used during the calibration procedure. As a proxy for the daily covariance employed in forecast evaluation, we use realized covariance computed as the sum of the cross-products of 15-min returns. The evaluation period spans from June 25, 2020, to June 28, 2024.

Forecast accuracy is assessed using the heteroscedasticity-adjusted mean squared error (HMSE). This metric accounts for heteroscedasticity, ensuring that forecast errors are comparable across periods of both high and low volatility.

The HMSE loss function is defined as:

$$\text{HMSE} = \sum_{m=1}^{M-1} \sum_{n=m+1}^M \text{HMSE}_{mn} \quad (36)$$

where the loss associated with each pair of instruments is given by

$$\text{HMSE}_{mn} = \frac{1}{T} \sum_{t=1}^T \left(\frac{\text{cov}_{F,mn,t} - \text{cov}_{R,mn,t}}{h_{mt}^{1/2} h_{nt}^{1/2}} \right)^2 \quad (37)$$

where $\text{cov}_{R,mn,t}$ denotes the realized covariance between instruments m and n , $\text{cov}_{F,mn,t}$ denotes the corresponding covariance forecast at time t , and T is the number of forecasts.

The statistical significance of the results is assessed using the Model Confidence Set (MCS) procedure proposed by Hansen et al. (2011). This procedure identifies the set of superior models at a given confidence level. The results of the joint evaluation across all assets are reported in Table 7.

Based on the HMSE criterion and the MCS test, the set of best-performing models consists exclusively of the proposed robust range-based DCC specifications. The two highest-ranked models are the BIP-cDCC-BM-RGARCH specification assuming a normal distribution with filtration levels of 0.90 and 0.99. This finding indicates that the covariance forecasts generated by the proposed models are more accurate than those produced by all benchmark specifications considered in the study, including the traditional DCC models, the standard range-based DCC model, and the standard robust DCC model. The least accurate forecasts are obtained from the traditional DCC specifications.

Robust DCC models estimated under the normal distribution generally outperform their counterparts based on the Student's t -distribution. In particular, the BIP-cDCC-BM-RGARCH specifications with a normal distribution consistently outperform the corresponding BIP-cDCC-BIP-RGARCH models estimated under the same distributional assumption. The results do not, however, identify a single clearly optimal filtration level, although a value of 0.99 appears to perform particularly well. At this filtration level, the models assuming a normal distribution rank second and third overall.

We next evaluate covariance forecasting performance separately for each asset pair. The results of this comparison are presented in Table 8.

The BIP-cDCC-BM-RGARCH model with a normal distribution and a filtration level of 0.90 – the best-performing specification in the joint evaluation – also produces the most accurate forecasts for the equities-Bitcoin, gold-equities, and oil-gold pairs. For all pairs involving Bitcoin (gold-Bitcoin, equities-Bitcoin, and oil-Bitcoin), only the proposed robust range-based DCC models are included in the set of superior models according to the MCS test. This superior performance is attributable to their lower sensitivity to outliers, which play a crucial role in the dynamics of Bitcoin prices.

For the gold-equities and oil-gold pairs, the set of superior models additionally includes the DCC-RGARCH specifications alongside the proposed models. The only exception is the oil-equities pair, for which the most accurate covariance forecasts are generated exclusively by the

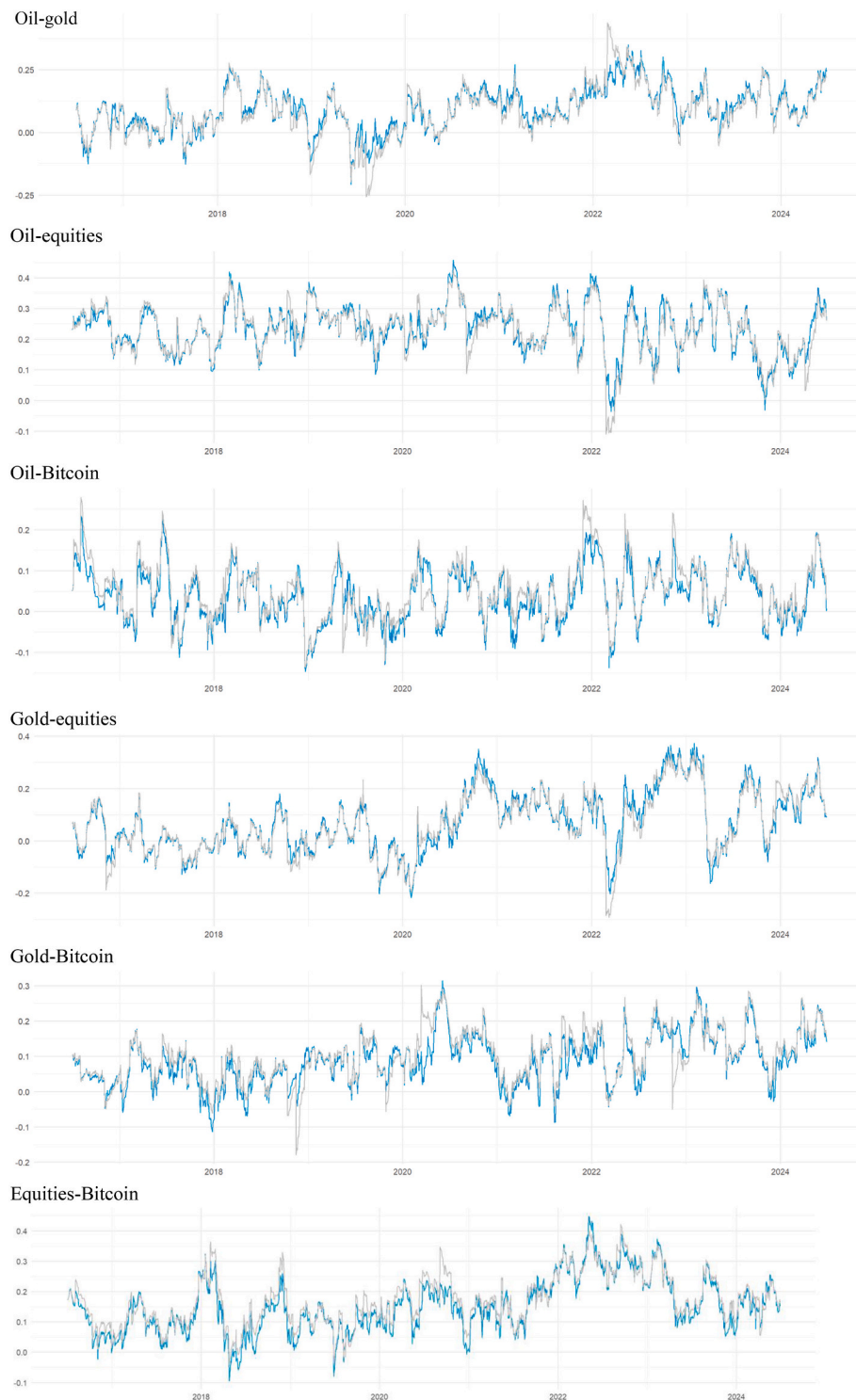


Fig. 2. Time-varying conditional correlations estimated by the DCC-RGARCH-n and BIP-cDCC-BM-RGARCH-n models.

Note: The blue line represents the BIP-cDCC-BM-RGARCH-n model with a filtration level of 0.9, while the gray line represents the benchmark DCC-RGARCH-n model based on the Garman-Klass estimator.

DCC-RGARCH models.

The remaining conclusions from the joint covariance forecast evaluation are broadly consistent with the results obtained for individual asset pairs. In particular, robust DCC models estimated under the normal distribution outperform those based on the Student's *t*-distribution, while the BIP-cDCC-BM-RGARCH specifications consistently outperform their BIP-cDCC-BIP-RGARCH counterparts.

7. Forecasting portfolio Value-at-Risk

The importance of accurate variance and covariance forecasts stems not only from their role as fundamental descriptive measures in financial markets but also from their wide range of practical applications. One of the most important applications is risk measurement. In this context, Value-at-Risk has become one of the most widely used risk metrics in modern finance. Its popularity is attributable to several practical

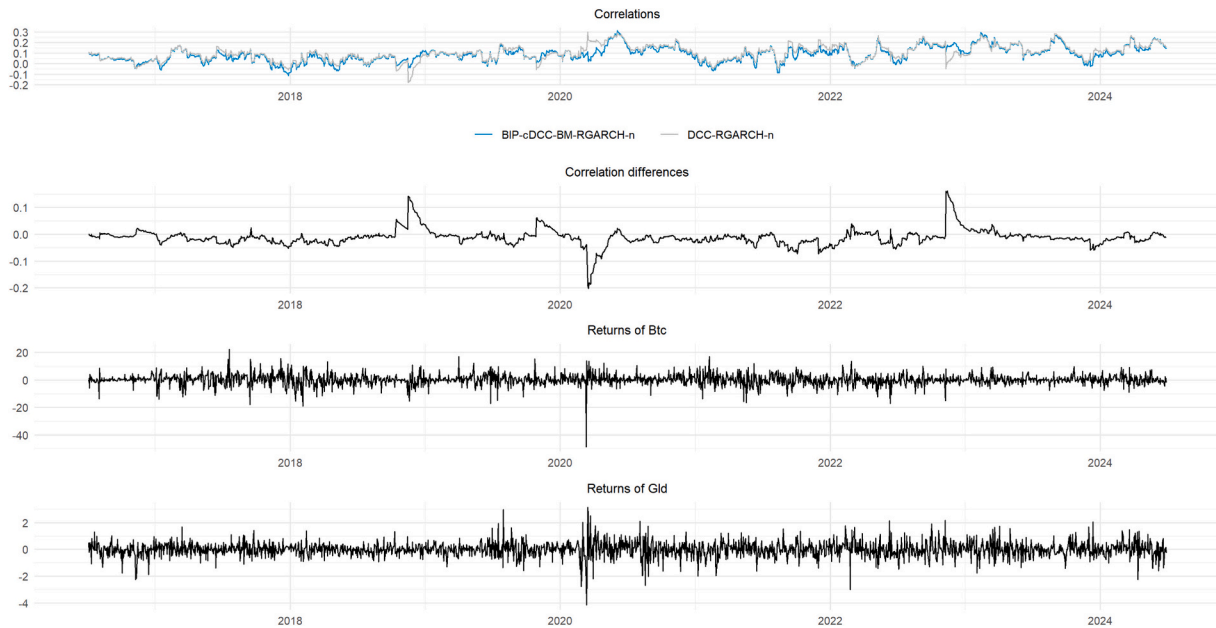


Fig. 3. Bitcoin return shocks and differences in conditional correlations between the DCC-RGARCH-n and BIP-cDCC-BM-RGARCH-n models for the Bitcoin-gold pair. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Note: Larger Bitcoin return shocks are generally associated with larger differences in conditional correlations, indicating a stronger impact of robust filtering during periods of extreme market movements.

advantages. VaR has an intuitive interpretation, provides concise information about the maximum expected loss under normal market conditions, and – particularly from a practical perspective – is readily applicable to complex portfolios. These features have contributed to its adoption as a standard risk measure regularly disclosed by major financial institutions.

VaR forecasting is subject to many of the same challenges as variance and covariance forecasting, particularly volatility clustering and the presence of extreme observations. Outliers may result in persistently inflated volatility estimates and, consequently, excessively conservative VaR forecasts that fail to revert to normal risk levels once the underlying shock dissipates. This issue is especially relevant during periods of market stress, when extreme returns occur more frequently. Consequently, the proper treatment of outliers is essential for reliable VaR prediction. At the same time, exploiting the additional information contained in daily high and low prices provides a more efficient measure of price variability. Combining robust estimation techniques with range-based volatility models therefore represents two complementary approaches that are particularly well suited to generating accurate and stable VaR forecasts. Accordingly, in this section, we use the previously obtained variance and covariance forecasts to generate VaR predictions for a portfolio composed of the assets under study.

The VaR forecasts are computed using the same dataset and evaluation period as those employed in the variance and covariance forecasting exercise. Specifically, we construct a portfolio consisting of oil, gold, equities, and Bitcoin over the period from June 25, 2020, to June 28, 2024. As in the previous section, an out-of-sample one-day-ahead rolling-window procedure is employed. The VaR confidence level is set at the conventional 5%, and an equally weighted portfolio allocation is assumed. VaR forecasts are generated for daily returns; intraday returns are not used in the evaluation procedure. VaR for the portfolio return r_t^p is defined as follows:

$$P(r_t^p < VaR_{t,\alpha}(r_t^p)) = \alpha \quad (38)$$

where α denotes the VaR level. This expression shows that VaR corresponds to the α -quantile of the portfolio return. To obtain VaR forecasts, we employ the semiparametric filtered historical simulation method of

Barone-Adesi et al. (1998), which combines parametric estimation of the conditional mean and variance with nonparametric estimation of the return quantile. The forecasts are obtained from the following expression:

$$\widehat{VaR}_{t,\alpha}(r_{t+1}^p) = \omega' \hat{\gamma}_{t+1} + \sqrt{\omega' \widehat{cov}_{t+1} \omega} \cdot \widehat{VaR}_{t,\alpha}(z_{t+1}^p) \quad (39)$$

where ω and $\hat{\gamma}_{t+1}$ denote the vectors of portfolio weights and mean forecasts, respectively, $\widehat{cov}_{t+1}$ denotes the predicted covariance matrix, and $\widehat{VaR}_{t,\alpha}(z_{t+1}^p)$ represents the predicted α -quantile of the standardized portfolio return. The key inputs determining the VaR forecasts – the conditional mean forecasts $\hat{\gamma}_{t+1}$ and the conditional covariance forecasts $\widehat{cov}_{t+1}$ – are derived from all previously considered volatility models. These include both the newly proposed multivariate robust RGARCH specifications and the conventional benchmark models, thereby enabling an assessment of the practical usefulness of the proposed approach in the context of VaR forecasting.

To evaluate the accuracy of the resulting VaR forecasts, we implement a two-step procedure consisting of statistical tests and loss-function analysis. The statistical evaluation comprises a set of tests designed to assess two fundamental criteria. The first is unconditional coverage, which examines whether the observed number of VaR exceedances is consistent with the assumed VaR level. This criterion is evaluated using the binomial test, which relies on the exact discrete distribution of exceedances, and the Kupiec test, which is based on an asymptotic chi-square approximation within a continuous-distribution framework (Kupiec, 1995). The second criterion is independence, which assesses whether VaR exceedances occur independently over time. This property is evaluated using the traditional Ljung–Box test, based on the autocorrelation function (Ljung & Box, 1978), and the generalized Markov test proposed by Pajhede (2017), which builds on the Markov-chain framework and extends the well-known test of Christoffersen (1998).

As indicated by the p -values of the VaR backtesting procedures reported in Table 9, the VaR forecasts generated by all considered models satisfy both evaluation criteria. Specifically, no model exhibits statistically significant deviations from the nominal VaR level in terms of

Table 7

Joint evaluation of covariance forecast accuracy based on HMSE and the MCS procedure.

Model	HMSE	MCS <i>p</i> -value	Rank
GARCH models			
DCC-n	1.695	0.000	
DCC-t	1.677	0.001	
cDCC-n	1.662	0.001	
cDCC-t	1.673	0.004	
RGARCH models			
DCC-n	1.254	0.001	
DCC-t	1.242	0.001	
Robust GARCH models, filtration 0.99			
BIP-cDCC-n	1.351	0.001	
BIP-cDCC-t	1.374	0.023	
Proposed models: robust RGARCH, filtration 0.99			
BIP-cDCC-n	1.119	0.105*	3
BIP-cDCC-t	1.170	0.105*	9
BIP*-cDCC-n	1.103	0.914*	2
BIP*-cDCC-t	1.206	0.023	
Robust GARCH models, filtration 0.975			
BIP-cDCC-n	1.387	0.000	
BIP-cDCC-t	1.400	0.001	
Proposed models: robust RGARCH, filtration 0.975			
BIP-cDCC-n	1.139	0.105*	6
BIP-cDCC-t	1.181	0.023	
BIP*-cDCC-n	1.125	0.105*	5
BIP*-cDCC-t	1.234	0.001	
Robust GARCH models, filtration 0.95			
BIP-cDCC-n	1.358	0.000	
BIP-cDCC-t	1.404	0.001	
Proposed models: robust RGARCH, filtration 0.95			
BIP-cDCC-n	1.147	0.105*	7
BIP-cDCC-t	1.170	0.023	
BIP*-cDCC-n	1.121	0.105*	4
BIP*-cDCC-t	1.252	0.004	
Robust GARCH models, filtration 0.9			
BIP-cDCC-n	1.352	0.000	
BIP-cDCC-t	1.453	0.001	
Proposed models: robust RGARCH, filtration 0.9			
BIP-cDCC-n	1.165	0.023	
BIP-cDCC-t	1.154	0.105*	8
BIP*-cDCC-n	1.103	1.000*	1
BIP*-cDCC-t	1.240	0.001	

Notes: For all RGARCH models, the Garman-Klass variance estimator is applied. HMSE is the sum of $HMSE_{mn}$ for all instrument pairs. BIP*-cDCC denotes the BIP-cDCC-BM-RGARCH model. The lowest value of the loss function is marked in bold. The symbol * indicates that the model belongs to the set of best models at the 0.90 confidence level. The ranking refers to models included in the set of best models. The evaluation period spans from June 25, 2020, to June 28, 2024.

exceedance frequency, and no evidence of dependence in VaR violations is detected. Accordingly, all models can be regarded as statistically acceptable according to the applied backtesting procedures.

The practical usefulness of the resulting risk forecasts is further evaluated using loss functions. For this purpose, we employ the regulatory loss function (RLF) proposed by [Sarma et al. \(2003\)](#), together with three additional regulatory loss functions - C1, C2, and C3 - introduced by [Caporin \(2008\)](#). These loss functions are defined as follows:

$$l_{t+1}^{RLF} = \begin{cases} (r_{t+1}^p - \widehat{VaR}_{t,\alpha}(r_{t+1}^p))^2, & \text{if } r_{t+1} < \widehat{VaR}_{t,\alpha}(r_{t+1}^p) \\ 0, & \text{otherwise,} \end{cases} \quad (40)$$

$$l_{t+1}^{C1} = \begin{cases} \left| 1 - \frac{r_{t+1}^p}{\widehat{VaR}_{t,\alpha}(r_{t+1}^p)} \right|, & \text{if } r_{t+1} < \widehat{VaR}_{t,\alpha}(r_{t+1}^p) \\ 0, & \text{otherwise,} \end{cases} \quad (41)$$

$$l_{t+1}^{C2} = \begin{cases} \frac{(|r_{t+1}^p| - |\widehat{VaR}_{t,\alpha}(r_{t+1}^p)|)^2}{|\widehat{VaR}_{t,\alpha}(r_{t+1}^p)|}, & \text{if } r_{t+1} < \widehat{VaR}_{t,\alpha}(r_{t+1}^p) \\ 0, & \text{otherwise,} \end{cases} \quad (42)$$

$$l_{t+1}^{C3} = \begin{cases} |r_{t+1}^p - \widehat{VaR}_{t,\alpha}(r_{t+1}^p)|, & \text{if } r_{t+1} < \widehat{VaR}_{t,\alpha}(r_{t+1}^p) \\ 0, & \text{otherwise.} \end{cases} \quad (43)$$

These loss functions measure the magnitude of losses incurred when the VaR threshold is breached. We consider them particularly relevant from a practical perspective because they penalize underestimated VaR forecasts, which may expose financial institutions to liquidity shortages. Moreover, they are closely aligned with regulatory requirements designed to safeguard the financial system against institutional insolvency.

To formally assess the relative performance of the models under consideration, we employ the MCS procedure. As in the covariance forecasting analysis, this approach enables the identification of the set of superior models at a given confidence level.

The mean values of the RLF, C1, C2, and C3 loss functions, together with the results of the MCS test and the rankings of models included in the set of superior models, are reported in [Table 10](#).

The rankings obtained across all loss functions consistently indicate that the best-performing specification belongs to the family of multivariate robust RGARCH models, thereby demonstrating the advantages of the approach proposed in this study. The highest rank under all evaluation criteria is achieved by the same specification, namely the BIP-cDCC-BIP-RGARCH model assuming a normal distribution and a filtration level of 0.975. Although the subsequent rankings vary across specifications and depend on the selected loss function, an important observation is that the highest-ranked models are consistently concentrated within the proposed model family. For each loss function, all models ranked within the top four positions belong exclusively to this group.

The MCS test further refines the conclusions regarding model performance by incorporating statistical significance into the evaluation. For two loss functions – RLF and C2 – the results are inconclusive because the performance differences across models are relatively small. Consequently, the resulting set of superior models is fairly broad and includes representatives of all model classes considered in the study. In

Table 8
Covariance forecast evaluation by asset pair based on HMSE and the MCS procedure.

Model	Gold-Bitcoin			Equities-Bitcoin			Oil-Bitcoin			Gold-equities			Oil-gold			Oil-equities		
	HMSE	MCS p-value	Rank	HMSE	MCS p-value	Rank	HMSE	MCS p-value	Rank	HMSE	MCS p-value	Rank	HMSE	MCS p-value	Rank	HMSE	MCS p-value	Rank
GARCH models																		
DCC-n	0.353	0.000		0.808	0.000		0.115	0.000		0.151	0.000		0.102	0.031		0.167	0.000	
DCC-t	0.332	0.000		0.786	0.000		0.124	0.000		0.163	0.000		0.105	0.007		0.167	0.000	
cDCC-n	0.330	0.000		0.806	0.000		0.104	0.000		0.152	0.000		0.103	0.014		0.168	0.000	
cDCC-t	0.324	0.000		0.794	0.001		0.116	0.000		0.164	0.000		0.104	0.014		0.170	0.000	
RGARCH models																		
DCC-n	0.286	0.000		0.494	0.011		0.105	0.000		0.133	0.861*	2	0.098	0.391*	6	0.137	1.000*	1
DCC-t	0.277	0.000		0.485	0.004		0.111	0.000		0.134	0.861*	3	0.096	0.594*	2	0.138	0.606*	2
Robust GARCH models, filtration 0.99																		
BIP-cDCC-n	0.284	0.000		0.521	0.006		0.103	0.000		0.164	0.000		0.105	0.007		0.175	0.000	
BIP-cDCC-t	0.277	0.002		0.518	0.033		0.103	0.000		0.178	0.000		0.110	0.006		0.188	0.000	
Proposed models: robust RGARCH, filtration 0.99																		
BIP-cDCC-n	0.248	0.072		0.395	0.128*	8	0.094	0.048		0.137	0.071		0.099	0.014		0.145	0.001	
BIP-cDCC-t	0.259	0.007		0.408	0.100		0.099	0.000		0.144	0.000		0.102	0.007		0.159	0.000	
BIP*-cDCC-n	0.245	1.000*	1	0.388	0.190*	5	0.093	0.930*	2	0.136	0.142*	6	0.099	0.014		0.141	0.006	
BIP*-cDCC-t	0.257	0.005		0.435	0.063		0.102	0.000		0.143	0.016		0.102	0.007		0.167	0.000	
Robust GARCH models, filtration 0.975																		
BIP-cDCC-n	0.313	0.000		0.517	0.009		0.099	0.000		0.168	0.000		0.106	0.007		0.183	0.000	
BIP-cDCC-t	0.304	0.000		0.504	0.021		0.099	0.000		0.181	0.000		0.112	0.007		0.201	0.000	
Proposed models: robust RGARCH, filtration 0.975																		
BIP-cDCC-n	0.266	0.002		0.396	0.100		0.094	0.703*	4	0.139	0.003		0.098	0.391*	5	0.148	0.001	
BIP-cDCC-t	0.272	0.000		0.403	0.129*	9	0.096	0.000		0.147	0.000		0.101	0.007		0.162	0.000	
BIP*-cDCC-n	0.258	0.005		0.396	0.100		0.094	0.028		0.135	0.287*	5	0.097	0.391*	4	0.145	0.000	
BIP*-cDCC-t	0.274	0.000		0.444	0.028		0.101	0.000		0.145	0.000		0.101	0.007		0.169	0.000	
Robust GARCH models, filtration 0.95																		
BIP-cDCC-n	0.309	0.000		0.479	0.021		0.097	0.000		0.172	0.000		0.107	0.007		0.194	0.000	
BIP-cDCC-t	0.307	0.000		0.486	0.021		0.098	0.000		0.187	0.000		0.115	0.006		0.211	0.000	
Proposed models: robust RGARCH, filtration 0.95																		
BIP-cDCC-n	0.273	0.000		0.393	0.129*	7	0.094	0.028		0.139	0.005		0.099	0.371*	7	0.150	0.000	
BIP-cDCC-t	0.278	0.000		0.384	0.538*	4	0.095	0.000		0.147	0.000		0.102	0.007		0.163	0.000	
BIP*-cDCC-n	0.268	0.001		0.380	0.538*	3	0.095	0.000		0.135	0.547*	4	0.097	0.391*	3	0.146	0.000	
BIP*-cDCC-t	0.285	0.000		0.450	0.047		0.101	0.000		0.145	0.000		0.101	0.014		0.170	0.000	
Robust GARCH models, filtration 0.9																		
BIP-cDCC-n	0.303	0.000		0.460	0.021		0.096	0.000		0.173	0.000		0.110	0.007		0.210	0.000	
BIP-cDCC-t	0.320	0.000		0.492	0.023		0.101	0.000		0.187	0.000		0.120	0.007		0.233	0.000	
Proposed models: robust RGARCH, filtration 0.9																		
BIP-cDCC-n	0.286	0.000		0.390	0.291*	6	0.095	0.001		0.141	0.001		0.100	0.031		0.154	0.000	
BIP-cDCC-t	0.274	0.005		0.369	0.825*	2	0.093	1.000*	1	0.147	0.000		0.103	0.007		0.169	0.000	
BIP*-cDCC-n	0.270	0.002		0.366	1.000*	1	0.093	0.930*	3	0.133	1.000*	1	0.096	1.000*	1	0.145	0.001	
BIP*-cDCC-t	0.287	0.000		0.434	0.049		0.103	0.000		0.143	0.000		0.100	0.014		0.173	0.000	

Notes: For all RGARCH models, the Garman-Klass variance estimator is applied. BIP*-cDCC denotes the BIP-cDCC-BM-RGARCH model. The lowest values of the loss function are marked in bold. The symbol * indicates that the model belongs to the set of best models at the 0.90 confidence level. The ranking refers to models included in the set of best models. The evaluation period spans from June 25, 2020, to June 28, 2024.

Table 9

Evaluation of VaR forecasts based on unconditional coverage and independence tests.

Model	Unconditional coverage tests				Independence tests			
	Binomial		Kupiec		LB		Generalized Markov	
	Stat.	P-value	Stat.	P-value	Stat.	P-value	Stat.	P-value
GARCH models								
DCC-n	46	0.289	0.425	0.514	1.118	0.952	1.596	0.206
DCC-t	52	0.376	0.050	0.824	4.889	0.430	2.997	0.083
cDCC-n	46	0.289	0.425	0.514	1.118	0.952	1.596	0.206
cDCC-t	51	0.431	0.006	0.937	3.706	0.592	2.413	0.120
RGARCH models								
DCC-n	48	0.397	0.127	0.721	2.023	0.846	1.770	0.183
DCC-t	53	0.323	0.134	0.715	2.261	0.812	0.748	0.387
Robust GARCH models, filtration 0.99								
BIP-cDCC-n	47	0.342	0.254	0.614	0.356	0.996	0.138	0.710
BIP-cDCC-t	48	0.397	0.127	0.721	3.603	0.608	0.911	0.340
Proposed models: robust RGARCH, filtration 0.99								
BIP-cDCC-n	47	0.342	0.254	0.614	2.967	0.705	1.229	0.268
BIP-cDCC-t	53	0.323	0.134	0.715	2.261	0.812	0.748	0.387
BIP*-cDCC-n	47	0.342	0.254	0.614	2.363	0.797	2.265	0.132
BIP*-cDCC-t	52	0.376	0.050	0.824	1.832	0.872	0.411	0.521
Robust GARCH models, filtration 0.975								
BIP-cDCC-n	46	0.289	0.425	0.514	0.385	0.996	0.290	0.590
BIP-cDCC-t	47	0.342	0.254	0.614	3.854	0.571	1.269	0.260
Proposed models: robust RGARCH, filtration 0.975								
BIP-cDCC-n	47	0.342	0.254	0.614	2.967	0.705	1.229	0.268
BIP-cDCC-t	50	0.512	0.004	0.948	3.253	0.661	1.040	0.308
BIP*-cDCC-n	47	0.342	0.254	0.614	2.363	0.797	2.265	0.132
BIP*-cDCC-t	52	0.376	0.050	0.824	1.832	0.872	0.411	0.521
Robust GARCH models, filtration 0.95								
BIP-cDCC-n	48	0.397	0.127	0.721	0.753	0.980	0.946	0.331
BIP-cDCC-t	49	0.454	0.044	0.833	3.498	0.624	1.338	0.247
Proposed models: robust RGARCH, filtration 0.95								
BIP-cDCC-n	48	0.397	0.127	0.721	2.024	0.846	1.818	0.178
BIP-cDCC-t	51	0.431	0.006	0.937	3.047	0.693	0.719	0.396
BIP*-cDCC-n	46	0.289	0.425	0.514	2.379	0.795	1.688	0.194
BIP*-cDCC-t	51	0.431	0.006	0.937	2.084	0.837	0.660	0.417
Robust GARCH models, filtration 0.9								
BIP-cDCC-n	47	0.342	0.254	0.614	1.528	0.910	0.606	0.436
BIP-cDCC-t	48	0.397	0.127	0.721	2.910	0.714	1.818	0.178
Proposed models: robust RGARCH, filtration 0.9								
BIP-cDCC-n	51	0.431	0.006	0.937	1.271	0.938	0.606	0.436
BIP-cDCC-t	52	0.376	0.050	0.824	2.885	0.718	1.818	0.178
BIP*-cDCC-n	47	0.342	0.254	0.614	2.363	0.797	0.689	0.406
BIP*-cDCC-t	51	0.431	0.006	0.937	2.084	0.837	0.458	0.498

Notes: For all RGARCH models, the Garman-Klass variance estimator is applied. BIP*-cDCC denotes the BIP-cDCC-BM-RGARCH model. The evaluation period spans from June 25, 2020, to June 28, 2024.

contrast, for the remaining loss functions – C1 and C3 – the differences in forecasting performance are sufficiently pronounced to allow statistically significant discrimination among models. In both cases, all specifications identified as superior belong to the family of multivariate robust RGARCH models. This finding provides additional empirical support for the proposed modeling framework.

With regard to the application of the proposed models to VaR forecasting, the analysis does not yield a definitive conclusion concerning the optimal filtration level. Although the best-performing specifications are generally associated with higher filtration thresholds, particularly 0.99 and 0.975, these differences are not statistically significant.

A clearer conclusion emerges with respect to the choice of

conditional distribution. Models estimated under the normal distribution perform significantly better than those based on the Student's *t*-distribution. Specifications assuming the Student's *t*-distribution either do not appear in the MCS set or rank substantially lower. This finding reinforces the earlier evidence from the covariance forecasting analysis and confirms the superiority of the normal distribution in the present setting.

Overall, the principal conclusion is the consistent and robust out-performance of the proposed models relative to the benchmark specifications in the context of VaR forecasting. This result highlights the practical value of the robust RGARCH framework and underscores its potential as a superior alternative to traditional volatility-modeling

Table 10

Evaluation of VaR forecasts based on the regulatory loss function RLF (Sarma et al., 2003), and the regulatory criteria C1, C2, and C3 (Caporin, 2008).

Model	Loss function RLF			Loss function C1			Loss function C2			Loss function C3		
	Function value	MCS p-value	Rank	Function value	MCS p-value	Rank	Function value	MCS p-value	Rank	Function value	MCS p-value	Rank
GARCH models												
DCC-n	0.057	0.397*	15	0.019	0.056		0.030	0.422*	14	0.036	0.018	
DCC-t	0.062	0.062		0.022	0.027		0.035	0.032		0.039	0.007	
cDCC-n	0.057	0.292*	17	0.019	0.027		0.030	0.310*	16	0.036	0.007	
cDCC-t	0.062	0.049		0.022	0.027		0.035	0.023		0.039	0.007	
RGARCH models												
DCC-n	0.056	0.470*	13	0.019	0.027		0.030	0.434*	12	0.036	0.007	
DCC-t	0.066	0.034		0.024	0.016		0.038	0.048		0.041	0.006	
Robust GARCH models, filtration 0.99												
BIP-cDCC-n	0.057	0.385*	14	0.019	0.027		0.030	0.417*	13	0.036	0.007	
BIP-cDCC-t	0.061	0.119*	21	0.022	0.027		0.034	0.096		0.039	0.007	
Proposed models: robust RGARCH, filtration 0.99												
BIP-cDCC-n	0.054	0.850*	3	0.018	0.362*	2	0.028	0.807*	3	0.035	0.379*	2
BIP-cDCC-t	0.063	0.140*	23	0.023	0.027		0.036	0.036		0.039	0.007	
BIP*-cDCC-n	0.055	0.474*	8	0.019	0.056	7	0.029	0.449*	7	0.035	0.046	
BIP*-cDCC-t	0.063	0.031		0.023	0.026		0.036	0.015		0.039	0.007	
Robust GARCH models, filtration 0.975												
BIP-cDCC-n	0.056	0.474*	10	0.019	0.136*	8	0.029	0.449*	8	0.035	0.144*	5
BIP-cDCC-t	0.059	0.267*	18	0.021	0.027		0.032	0.189*	18	0.038	0.007	
Proposed models: robust RGARCH, filtration 0.975												
BIP-cDCC-n	0.054	1.000*	1	0.018	1.000*	1	0.028	1.000*	1	0.035	1.000*	1
BIP-cDCC-t	0.063	0.194*	22	0.023	0.027		0.036	0.125*	21	0.039	0.007	
BIP*-cDCC-n	0.055	0.639*	6	0.018	0.136*	6	0.029	0.627*	6	0.035	0.144*	5
BIP*-cDCC-t	0.063	0.071		0.023	0.027		0.036	0.017		0.039	0.007	
Robust GARCH models, filtration 0.95												
BIP-cDCC-n	0.056	0.639*	9	0.019	0.136*	9	0.029	0.708*	9	0.036	0.144*	6
BIP-cDCC-t	0.057	0.474*	16	0.021	0.027		0.031	0.422*	17	0.037	0.007	
Proposed models: robust RGARCH, filtration 0.95												
BIP-cDCC-n	0.054	0.850*	2	0.018	0.339*	3	0.028	0.807*	2	0.035	0.379*	3
BIP-cDCC-t	0.061	0.300*	20	0.022	0.027		0.035	0.220*	20	0.038	0.007	
BIP*-cDCC-n	0.055	0.690*	4	0.018	0.339*	4	0.028	0.727*	4	0.035	0.379*	4
BIP*-cDCC-t	0.063	0.090		0.023	0.027		0.036	0.020		0.039	0.007	
Robust GARCH models, filtration 0.9												
BIP-cDCC-n	0.056	0.639*	11	0.019	0.027		0.029	0.807*	11	0.036	0.007	
BIP-cDCC-t	0.056	0.819*	12	0.021	0.027		0.030	0.627*	15	0.037	0.007	
Proposed models: robust RGARCH, filtration 0.9												
BIP-cDCC-n	0.055	0.639*	7	0.019	0.030		0.029	0.449*	10	0.036	0.018	
BIP-cDCC-t	0.060	0.397*	19	0.022	0.027		0.034	0.317*	19	0.038	0.007	
BIP*-cDCC-n	0.055	0.639*	5	0.018	0.339*	5	0.028	0.708*	5	0.035	0.348*	6
BIP*-cDCC-t	0.063	0.053		0.023	0.027		0.036	0.025		0.039	0.007	

Notes: For all RGARCH models, the Garman–Klass variance estimator is used. BIP*-cDCC denotes the BIP-cDCC-BM-RGARCH model. The lowest loss values are highlighted in bold. An asterisk * indicates inclusion in the MCS at the 90% confidence level. Rankings are reported only for models included in the MCS. The evaluation period spans June 25, 2020 – June 28, 2024.

approaches in risk-management applications.

8. Robustness check using the Parkinson variance estimator

The proposed models, BIP-cDCC-BIP-RGARCH and BIP-cDCC-BM-RGARCH, are based on the Garman–Klass variance estimator (see Eqs. (25) and (32)), which is constructed using daily open, high, low, and

close prices. However, because the opening price may be sensitive to manipulation in periods of low liquidity and may not fully reflect market equilibrium, we additionally perform a robustness check using the Parkinson estimator (Parkinson, 1980), an alternative range-based variance measure that relies exclusively on high and low prices $\sigma_{P,n}^2 = [\ln(H_{nt}/L_{nt})]^2 / (4\ln 2)$. The Parkinson estimator has previously been employed in both the DCC-RGARCH model of Fiszeder et al. (2019)

Table 11

Estimation results for the correlation component using the Parkinson variance estimator in RGARCH models.

Model	ζ_1 (SE(ζ_1))	θ_1 (SE(θ_1))
GARCH models		
DCC-n	0.019 (0.002)	0.954 (0.017)
DCC-t	0.014 (0.003)	0.971 (0.031)
cDCC-n	0.020 (0.002)	0.953 (0.016)
cDCC-t	0.016 (0.004)	0.966 (0.038)
RGARCH models		
DCC-n	0.021 (0.002)	0.950 (0.016)
DCC-t	0.016 (0.004)	0.963 (0.031)
Robust GARCH models, filtration 0.99		
BIP-cDCC-n	0.020 (0.016)	0.952 (0.116)
BIP-cDCC-t	0.016 (0.019)	0.966 (0.155)
Proposed models: robust RGARCH, filtration 0.99		
BIP-cDCC-n	0.022 (0.019)	0.950 (0.145)
BIP-cDCC-t	0.017 (0.020)	0.961 (0.141)
BIP*-cDCC-n	0.023 (0.014)	0.945 (0.092)
BIP*-cDCC-t	0.018 (0.016)	0.959 (0.113)
Robust GARCH models, filtration 0.975		
BIP-cDCC-n	0.020 (0.014)	0.955 (0.100)
BIP-cDCC-t	0.016 (0.017)	0.969 (0.123)
Proposed models: robust RGARCH, filtration 0.975		
BIP-cDCC-n	0.023 (0.013)	0.947 (0.089)
BIP-cDCC-t	0.017 (0.011)	0.961 (0.074)
BIP*-cDCC-n	0.023 (0.016)	0.944 (0.108)
BIP*-cDCC-t	0.017 (0.022)	0.959 (0.155)
Robust GARCH models, filtration 0.95		
BIP-cDCC-n	0.020 (0.015)	0.958 (0.112)
BIP-cDCC-t	0.017 (0.017)	0.965 (0.122)
Proposed models: robust RGARCH, filtration 0.95		
BIP-cDCC-n	0.022 (0.015)	0.952 (0.112)
BIP-cDCC-t	0.016 (0.019)	0.968 (0.129)
BIP*-cDCC-n	0.023 (0.015)	0.945 (0.104)
BIP*-cDCC-t	0.017 (0.017)	0.960 (0.114)
Robust GARCH models, filtration 0.9		
BIP-cDCC-n	0.021 (0.015)	0.954 (0.106)
BIP-cDCC-t	0.017 (0.018)	0.969 (0.121)
Proposed models: robust RGARCH, filtration 0.9		
BIP-cDCC-n	0.022 (0.015)	0.955 (0.108)
BIP-cDCC-t	0.017 (0.019)	0.965 (0.145)
BIP*-cDCC-n	0.023 (0.016)	0.943 (0.105)
BIP*-cDCC-t	0.018 (0.018)	0.960 (0.129)

Notes: BIP*-cDCC denotes the BIP-cDCC-BM-RGARCH model. Standard errors are reported in parentheses. The estimation sample spans July 1, 2016 – June 28, 2024.

and the BM-RGARCH model of Fiszeder et al. (2024).

We begin by estimating the parameters of all thirty models; the results are reported in Table 11. The parameter estimates obtained for the proposed models are very similar to those presented in Table 6 in Section 5. The principal findings obtained using the Garman–Klass variance estimator remain unchanged when the Parkinson estimator is employed. Across all models, the estimates of ζ_1 are lower and those of θ_1 are higher under the conditional Student's t-distribution than under the conditional normal distribution. In addition, ζ_1 estimates tend to be higher and θ_1 estimates lower in DCC-RGARCH models than in DCC-GARCH models. In most cases, the proposed robust DCC-RGARCH specifications produce slightly higher ζ_1 estimates and slightly lower θ_1 estimates than the

Table 12

Joint evaluation of covariance forecast accuracy using the Parkinson variance estimator.

Model	HMSE	MCS p-value	Rank
GARCH models			
DCC-n	1.695	0.000	
DCC-t	1.677	0.000	
cDCC-n	1.662	0.000	
cDCC-t	1.673	0.000	
RGARCH models			
DCC-n	1.354	0.029	
DCC-t	1.306	0.004	
Robust GARCH models, filtration 0.99			
BIP-cDCC-n	1.351	0.021	
BIP-cDCC-t	1.374	0.053	
Proposed models: robust RGARCH, filtration 0.99			
BIP-cDCC-n	1.156	1.000*	1
BIP-cDCC-t	1.204	0.102*	8
BIP*-cDCC-n	1.159	0.820*	2
BIP*-cDCC-t	1.277	0.071	
Robust GARCH models, filtration 0.975			
BIP-cDCC	1.387	0.000	
BIP-cDCC	1.400	0.025	
Proposed models: robust RGARCH, filtration 0.975			
BIP-cDCC	1.175	0.304*	6
BIP-cDCC	1.212	0.099	
BIP*-cDCC	1.162	0.820*	4
BIP*-cDCC	1.291	0.058	
Robust GARCH models, filtration 0.95			
BIP-cDCC	1.358	0.000	
BIP-cDCC	1.404	0.005	
Proposed models: robust RGARCH, filtration 0.95			
BIP-cDCC	1.198	0.071	
BIP-cDCC	1.197	0.102*	7
BIP*-cDCC	1.170	0.597*	5
BIP*-cDCC	1.299	0.071	
Robust GARCH models, filtration 0.9			
BIP-cDCC	1.352	0.001	
BIP-cDCC	1.453	0.001	
Proposed models: robust RGARCH, filtration 0.9			
BIP-cDCC	1.231	0.058	
BIP-cDCC	1.215	0.071	
BIP*-cDCC	1.162	0.820*	3
BIP*-cDCC	1.273	0.058	

Notes: For all RGARCH models, the Parkinson variance estimator is applied. HMSE is the sum of $HMSE_{mn}$ for all instrument pairs. BIP*-cDCC denotes the BIP-cDCC-BM-RGARCH model. The lowest value of the loss function is marked in bold. The symbol * indicates that the model belongs to the set of best models at the 0.90 confidence level. The ranking refers to models included in the set of best models. The evaluation period spans from June 25, 2020, to June 28, 2024.

standard DCC-RGARCH models.

Next, we compare the forecasting performance of all models under consideration. The results of the joint evaluation across all assets are reported in Table 12, whereas the results for individual asset pairs are presented in Table 13.

The joint evaluation based on the Parkinson estimator, the HMSE criterion, and the MCS test confirms the main findings previously obtained using the Garman–Klass estimator. Specifically, the set of best-

Table 13

Covariance forecast evaluation by asset pair using the Parkinson variance estimator.

Model	Gold-Bitcoin			Equities-Bitcoin			Oil-Bitcoin			Gold-equities			Oil gold			Oil-equities		
	HMSE	MCS p-value	Rank	HMSE	MCS p-value	Rank	HMSE	MCS p-value	Rank	HMSE	MCS p-value	Rank	HMSE	MCS p-value	Rank	HMSE	MCS p-value	Rank
GARCH models																		
DCCn	0.353	0.000		0.808	0.000		0.115	0.000		0.151	0.006		0.102	0.007		0.167	0.000	
DCCt	0.332	0.000		0.786	0.000		0.124	0.000		0.163	0.002		0.105	0.001		0.167	0.000	
cDCCn	0.330	0.000		0.806	0.000		0.104	0.000		0.152	0.006		0.103	0.002		0.168	0.000	
cDCCt	0.324	0.000		0.794	0.000		0.116	0.000		0.164	0.006		0.104	0.001		0.170	0.000	
RGARCH models																		
DCCn	0.311	0.000		0.553	0.004		0.112	0.000		0.139	1.000*	1	0.099	0.434*	5	0.140	1.000*	1
DCCt	0.288	0.004		0.524	0.000		0.114	0.000		0.140	0.383*	2	0.098	0.653*	2	0.141	0.244*	2
Robust GARCH models, filtration 0.99																		
BIP-cDCC	0.284	0.001		0.521	0.006		0.103	0.000		0.164	0.001		0.105	0.000		0.175	0.000	
BIP-cDCC	0.277	0.077		0.518	0.020		0.103	0.000		0.178	0.006		0.110	0.000		0.188	0.000	
Proposed models: robust RGARCH, filtration 0.99																		
BIP-cDCC	0.257	1.000*	1	0.413	0.387*	9	0.095	0.484*	9	0.144	0.025		0.101	0.007		0.147	0.051	
BIP-cDCC	0.264	0.330*	3	0.423	0.314*	12	0.099	0.000		0.152	0.006		0.103	0.000		0.162	0.000	
BIP*-cDCC	0.258	0.686*	2	0.418	0.314*	10	0.096	0.011	10	0.143	0.221*	6	0.101	0.007		0.143	0.107*	3
BIP*-cDCC	0.268	0.139*	4	0.481	0.020		0.103	0.000		0.153	0.006		0.103	0.000		0.169	0.000	
Robust GARCH models, filtration 0.975																		
BIP-cDCC	0.313	0.000		0.517	0.006		0.099	0.000		0.168	0.001		0.106	0.000		0.183	0.000	
BIP-cDCC	0.304	0.000		0.504	0.010		0.099	0.000		0.181	0.005		0.112	0.000		0.201	0.000	
Proposed models: robust RGARCH, filtration 0.975																		
BIP-cDCC	0.276	0.017		0.408	0.618*	5	0.094	0.733*	4	0.146	0.006		0.100	0.124*	6	0.151	0.000	
BIP-cDCC	0.280	0.001		0.411	0.582*	7	0.097	0.000		0.153	0.005		0.104	0.000		0.167	0.000	
BIP*-cDCC	0.272	0.036		0.408	0.571*	6	0.095	0.484*	8	0.142	0.286*	5	0.099	0.297*	4	0.146	0.003	
BIP*-cDCC	0.285	0.001		0.479	0.020		0.103	0.000		0.153	0.006		0.102	0.001		0.170	0.000	
Robust GARCH models, filtration 0.95																		
BIP-cDCC	0.309	0.000		0.479	0.007		0.097	0.010		0.172	0.001		0.107	0.000		0.194	0.000	
BIP-cDCC	0.307	0.000		0.486	0.008		0.098	0.000		0.187	0.003		0.115	0.000		0.211	0.000	
Proposed models: robust RGARCH, filtration 0.95																		
BIP-cDCC	0.291	0.001		0.411	0.361*	8	0.094	0.596*	5	0.146	0.009		0.101	0.007		0.155	0.000	
BIP-cDCC	0.281	0.002		0.396	0.977*	2	0.093	0.733*	2	0.152	0.006		0.104	0.000		0.171	0.000	
BIP*-cDCC	0.283	0.001		0.406	0.802*	4	0.095	0.484*	7	0.142	0.286*	4	0.098	0.434*	3	0.146	0.006	
BIP*-cDCC	0.291	0.001		0.481	0.022		0.102	0.000		0.153	0.006		0.101	0.003		0.170	0.000	
Robust GARCH models, filtration 0.9																		
BIP-cDCC	0.303	0.001		0.460	0.019		0.096	0.134*	11	0.173	0.002		0.110	0.000		0.210	0.000	
BIP-cDCC	0.320	0.000		0.492	0.015		0.101	0.000		0.187	0.006		0.120	0.000		0.233	0.000	
Proposed models: robust RGARCH, filtration 0.9																		
BIP-cDCC	0.303	0.000		0.423	0.270*	11	0.094	0.733*	3	0.147	0.038		0.102	0.002		0.163	0.000	
BIP-cDCC	0.281	0.036		0.401	0.914*	3	0.093	1.000*	1	0.155	0.006		0.105	0.000		0.180	0.000	
BIP*-cDCC	0.289	0.001		0.395	1.000*	1	0.094	0.733*	6	0.141	0.383*	3	0.098	1.000*	1	0.146	0.019	
BIP*-cDCC	0.291	0.017		0.457	0.095		0.100	0.000		0.151	0.006		0.101	0.020		0.173	0.000	

Notes: For all RGARCH models, the Parkinson variance estimator is applied. BIP*-cDCC denotes the BIP-cDCC-BM-RGARCH model. The lowest values of the loss function are marked in bold. The symbol * indicates that the model belongs to the set of best models at the 0.90 confidence level. The ranking refers to models included in the set of best models. The evaluation period spans from June 25, 2020, to June 28, 2024.

performing models consists exclusively of the proposed robust range-based DCC specifications. Within this group, the BIP-cDCC-BIP-RGARCH and BIP-cDCC-BM-RGARCH models, both estimated under the normal distribution with a filtration level of 0.99, achieve the highest rankings.

The analysis further supports the conclusion that robust DCC models estimated under the normal distribution outperform those based on the Student's *t*-distribution. The results regarding the filtration level are also consistent with the earlier findings, indicating a clear preference for the highest level of 0.99, at which the models assuming normality rank first and second. With respect to the choice between the BIP-cDCC-BM-RGARCH and BIP-cDCC-BIP-RGARCH specifications, the BM-RGARCH models generally outperform their counterparts, in line with the previous results.

The disaggregated analysis at the level of individual asset pairs reveals that the proposed robust RGARCH models deliver the most accurate forecasts for four combinations: gold-Bitcoin, equities-Bitcoin, oil-Bitcoin, and oil-gold. This finding is consistent with the earlier observation that, for all pairs involving Bitcoin, only the proposed robust range-based DCC models are included in the MCS set of superior models. These results highlight the importance of the reduced sensitivity to outliers offered by the proposed specifications, particularly given the pronounced volatility of Bitcoin prices. Consistent with the earlier evidence, the only models outside the proposed family that appear in the set of superior models are the DCC-RGARCH specifications. These models perform best for the gold-equities and oil-equities pairs.

The conclusion that the normal distribution provides a more suitable specification than the Student's *t*-distribution remains valid for most asset pairs. By contrast, the preferred filtration level and the choice between the BIP-RGARCH and BM-RGARCH specifications vary across asset pairs.

As an additional robustness check, we examine the performance of the Rogers–Satchell range-based estimator (Rogers & Satchell, 1991), which relaxes the zero-drift assumption underlying the Parkinson and Garman–Klass estimators. The comparison is conducted using mean squared error (MSE) measures across the analyzed assets.

The results indicate that the Rogers–Satchell estimator generally produces larger estimation errors than the Garman–Klass and Parkinson estimators in our sample. This finding is consistent with the existing literature, which suggests that although the Rogers–Satchell estimator is robust to non-zero drift, it is typically less statistically efficient than the Garman–Klass estimator under standard market conditions. For brevity, the detailed results are reported in Table B.1 in Appendix B.

9. Discussion and directions for future research

9.1. Empirical scope of the analysis

The empirical analysis presented in this study focuses on assets representing several important segments of global financial markets, including commodities, equities, and cryptocurrencies. This heterogeneous set of assets provides a useful framework for examining volatility dynamics and cross-market interactions in environments characterized by diverse market structures, liquidity conditions, and exposure to macroeconomic shocks. Such settings are particularly relevant for evaluating multivariate volatility models, as investors increasingly

construct diversified portfolios spanning both traditional financial instruments and emerging digital assets.

At the same time, the analysis should be regarded as an empirical illustration rather than a comprehensive investigation covering all asset classes and market conditions. Future research could extend the analysis to a broader range of financial assets, including fixed-income securities, alternative commodities, and sector-specific equity indices. Another promising avenue would be to examine the performance of the proposed models in other financial applications, such as portfolio optimization, dynamic hedging, and systemic risk monitoring.

9.2. Robust modeling in the presence of extreme observations

The results highlight the importance of robust modeling techniques when analyzing financial data that are frequently affected by extreme observations and structural instability. Financial markets often experience abrupt price movements driven by macroeconomic announcements, geopolitical events, or shifts in investor sentiment. In such environments, volatility models that combine robustness to outliers with information-rich volatility measures may provide a more reliable representation of risk dynamics than conventional approaches based solely on closing prices.

9.3. Relationship to heavy-tailed and high-frequency volatility models

An important consideration concerns the relationship between the proposed framework and alternative approaches designed to capture non-Gaussian features of financial returns. Multivariate GARCH models with a conditional Student's *t*-distribution represent a widely used approach for accommodating heavy tails in return distributions. In the empirical analysis presented in this paper, models assuming both Gaussian and Student's *t*-distributions are estimated to account for this characteristic. The proposed framework therefore complements, rather than replaces, heavy-tailed specifications by addressing a different source of model instability, namely the influence of extreme observations on volatility dynamics.

Another important class of models relies on realized volatility or realized covariance measures constructed from high-frequency intraday data. Although such approaches may provide statistically more efficient volatility estimators, their implementation requires detailed intraday price information and involves additional challenges related to market microstructure noise and irregular trading. By contrast, range-based volatility measures extract substantially more information than closing prices while relying only on daily OHLC data. Consequently, the proposed models may be particularly useful in empirical settings where high-frequency data are unavailable or costly to process.

9.4. Limitations of range-based variance estimators in the presence of jumps

An important limitation concerns the properties of classical range-based variance estimators in the presence of discontinuous price movements. Estimators based on high–low ranges, such as those developed by Parkinson (1980) and Garman and Klass (1980), are derived under the assumption of continuous diffusion processes. In financial markets characterized by jumps, overnight gaps, or abrupt

news-driven price adjustments, these estimators may become biased because such price movements are not fully consistent with the underlying diffusion framework.

Although the robust filtering mechanisms incorporated into the proposed models help mitigate the influence of extreme observations on volatility dynamics, they do not directly address structural biases associated with jump components in asset prices. Consequently, the proposed framework should be interpreted primarily as a robust extension of range-based volatility modeling rather than a comprehensive solution to jump-related measurement issues.

Future research could therefore investigate whether jump-robust range-based estimators or bipower-type measures can be integrated into the multivariate RGARCH framework. Such extensions could combine the informational advantages of range-based volatility measures with improved robustness to discontinuous price movements.

9.5. Theoretical properties and future methodological work

From a theoretical perspective, the proposed models build upon several well-established components of the multivariate volatility literature. The dynamic correlation structure follows the corrected DCC framework introduced by Aielli (2013), while the robust filtering mechanisms are based on the bounded innovation propagation approach developed by Boudt et al. (2013) and the robust estimation framework of Muler and Yohai (2008). The univariate volatility equations extend range-based RGARCH specifications through the incorporation of robust weighting schemes. As a result, the proposed framework can be viewed as an integration of existing modeling components rather than the introduction of an entirely new stochastic structure.

Nevertheless, the empirical calibration of the robust weighting constants and the use of range-based volatility measures introduce additional elements whose implications for stationarity, ergodicity, and estimator consistency warrant further investigation. In particular, the impact of empirically calibrated thresholds on asymptotic properties has not yet been fully explored in the literature.

A complete theoretical treatment of these issues would require a detailed probabilistic analysis of the joint dynamics of the range-based volatility equations and the robust filtering mechanism. Such an analysis is beyond the scope of the present empirical study. Future research could therefore focus on establishing sufficient conditions for covariance stationarity and identification, as well as examining the asymptotic properties of the associated estimators.

10. Conclusions

This study examined the relationships among financial assets, with particular emphasis on the links between crude oil price fluctuations and developments in the gold, equity, and cryptocurrency markets. To model and forecast covariances within a portfolio composed of these assets, multivariate GARCH models were employed. The analysis focused on two features that are particularly important for modeling highly volatile financial markets: (1) robust estimation techniques and (2) range-based volatility measures.

The main methodological contribution of the study is the integration

of these two features within a unified multivariate framework. Specifically, the proposed models extend the standard cDCC specification by incorporating range-based volatility dynamics following Fiszeder et al. (2019), while simultaneously introducing robustness to extreme observations. Two alternative robust estimation approaches are considered: one based on the bounded innovation propagation (BIP) procedure of Boudt et al. (2013) and another based on the bounded M-type variance estimator developed by Muler and Yohai (2008).

The empirical results demonstrate that the proposed robust multivariate range-based GARCH models generate more accurate covariance forecasts than several benchmark specifications. These benchmarks include the standard DCC and cDCC models, the range-based DCC-RGARCH models, and the robust BIP-cDCC models. The practical value of the proposed framework is further supported by the superior Value-at-Risk forecasting performance obtained for a portfolio consisting of ETFs and Bitcoin representing the oil, gold, equity, and cryptocurrency markets.

Overall, the findings underscore the practical relevance of the robust RGARCH framework for modeling volatility spillovers and tail risks across financial and commodity markets. From a financial perspective, the results highlight the benefits of combining robustness to extreme observations with information-rich volatility measures when modeling risk in diversified portfolios. The proposed models improve the reliability of covariance and VaR forecasts, which are important inputs for portfolio allocation, risk management, and stress-testing applications. These findings are particularly relevant in periods of elevated market uncertainty and increasing integration between traditional financial assets and cryptocurrencies.

The superior Value-at-Risk forecasting performance of the proposed models also has important economic implications. More accurate VaR forecasts enable financial institutions and investors to assess portfolio risk more precisely, thereby improving capital allocation, risk control, and hedging decisions. Consequently, the proposed framework may offer tangible benefits for portfolio risk management relative to conventional multivariate volatility specifications.

CRediT authorship contribution statement

Piotr Fiszeder: Conceptualization, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Supervision, Validation, Writing – original draft, Writing – review & editing. **Marta Malecka:** Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review & editing.

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Appendix A

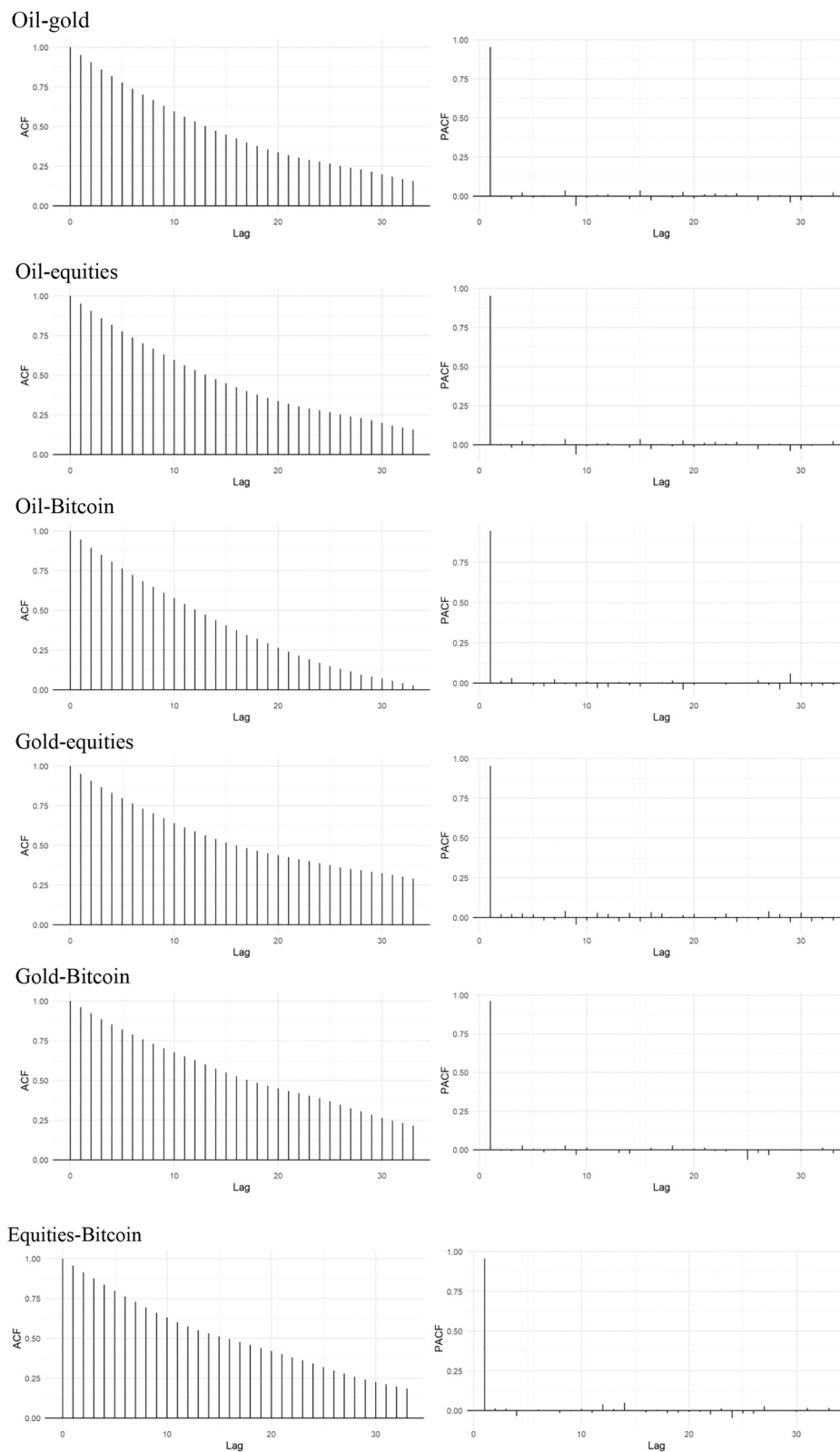


Fig. A.1. Correlograms of correlation differences between BIP-cDCC-BM-RGARCH-n model and the benchmark DCC-RGARCH-n model.

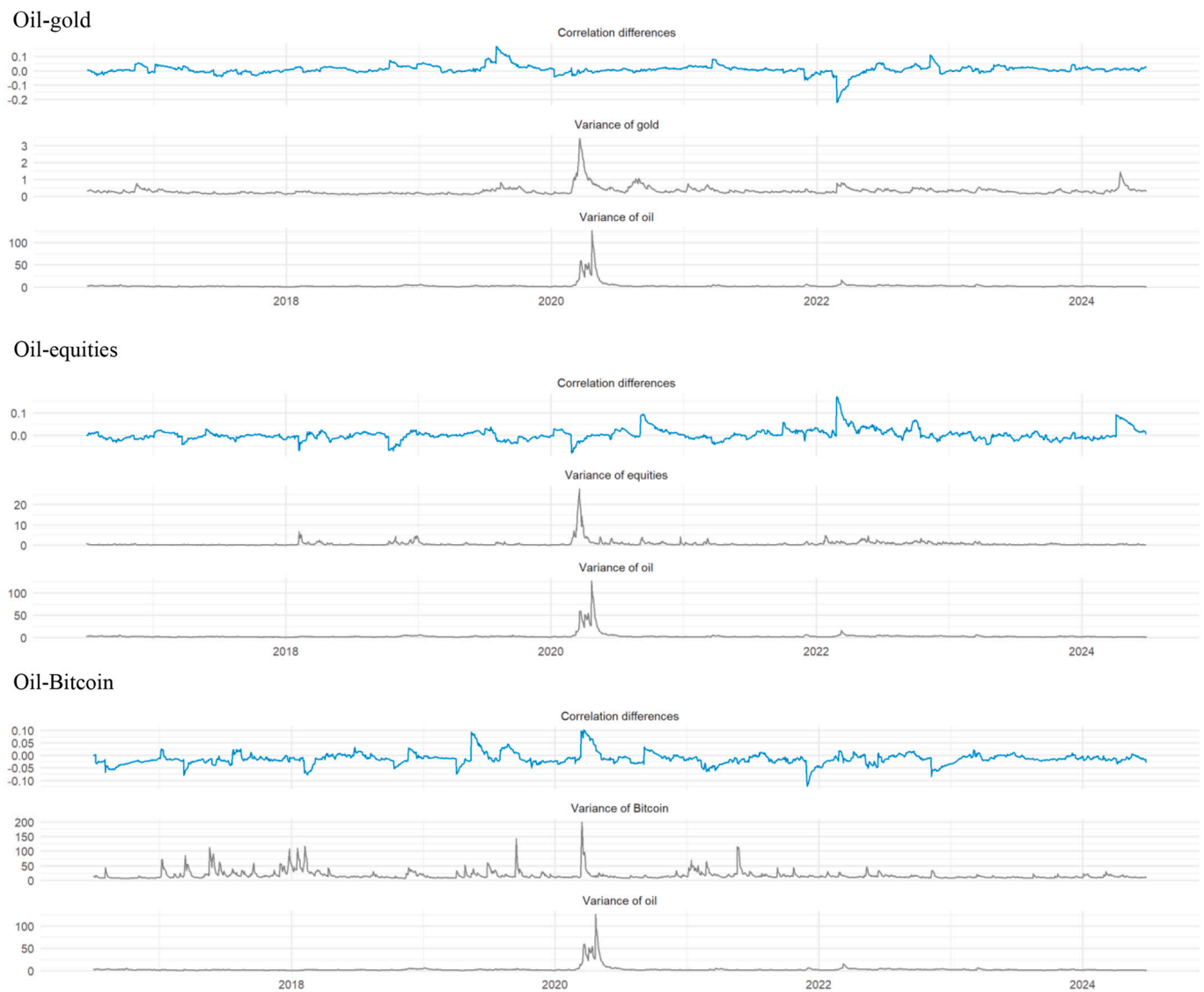
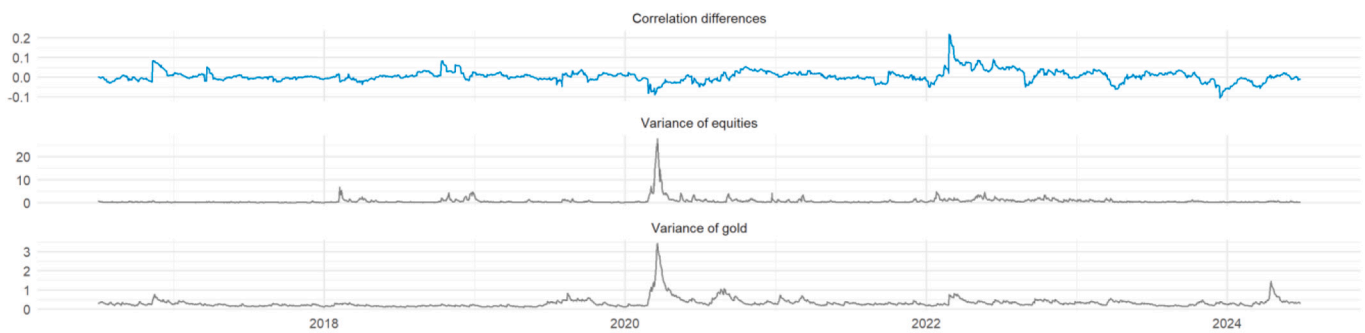
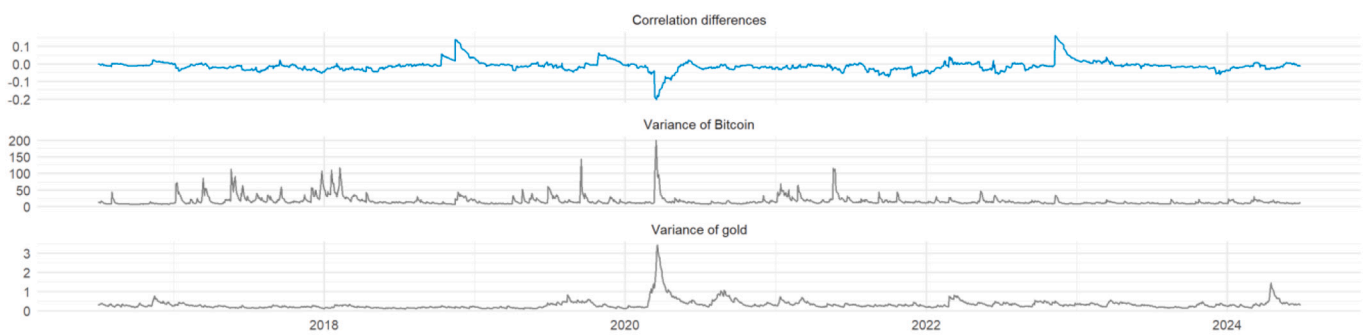


Fig. A.2. Differences between time-varying conditional correlations estimated using the BIP-cDCC-BM-RGARCH-n model and the benchmark DCC-RGARCH-n model.

Gold-equities



Gold-Bitcoin



Equities-Bitcoin

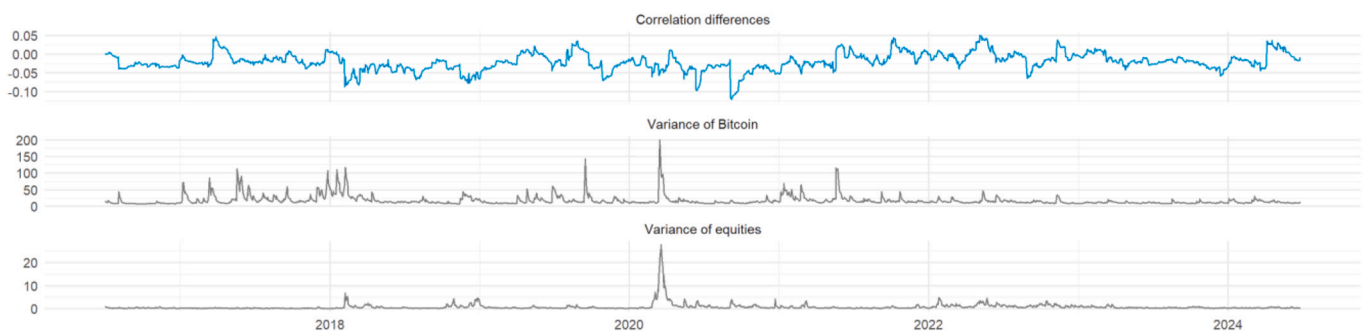


Fig. A.2. (continued).

Appendix B

Table B.1

Mean squared errors of alternative range-based volatility estimators.

Assets	Garman-Klass	Parkinson	Rogers-Satchell
Oil	44.535	24.268	61.448
Gold	0.068	0.079	0.112
Equities	0.531	0.701	0.922
Bitcoin	1870.117	1636.609	2843.171

Notes: The table reports mean squared errors (MSEs) for the Garman–Klass, Parkinson, and Rogers–Satchell estimators. In the analyzed sample, the Rogers–Satchell estimator generally produces the largest estimation errors.

Data availability

The data is available at the following address: <https://doi.org/10.18150/AN5NET>.

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